## Exponential Functions (1:08:22)

## Intro (0:00 to 6:52)

Express the number " e " to 6 decimal places.

Describe the steps one would take to express "e" on the TI-89 calculator.

Describe the process of negative exponentiation.

Solve for the following expressions using negative exponentiation:
$3^{-1}$
$2.7^{-1}$
$2.72^{-1}$
$2.718^{-1}$
$2.7183^{-1}$
$2.71828^{-1}$

## Part 1: $y(x)=e^{-x}(6: 52$ to 14:32)

Evaluate the function $y(x)=e^{-x}$ when $x=0,1,2,3,4,5$. Express your answer to 3 decimal places. Plot these values on a graph.

Describe the assumption made about the function $y(x)=e^{-x}$ for $x$ values 5 or greater.

Determine which $x$ value satisfies the condition $0.5=\mathrm{e}^{-x}$

Describe how to perform the natural log, In, operation on the TI-89 calculator.

Evaluate the function $y(x)=e^{-x}$ when $x=1.5,2.5,4.3$. Express your answer to 3 decimal places.

Use the In operation to determine which $x$ values satisfies these conditions:

$$
\begin{aligned}
& .75=e^{-x} \\
& .25=e^{-x} \\
& .125=e^{-x}
\end{aligned}
$$

Part 2: $y(x)=A e^{-x}$ (14:32 to 18:20)

Evaluate the function $y(x)=6 e^{-x}$ when $x=1.9$ Express your answer to 3 decimal places.

Determine the $x$ value that satisfies the condition $5=6 e^{-x}$

Evaluate these expressions at the specified $x$ value.

$$
\begin{aligned}
& y(x)=12 e^{-x} \text { when } x=0.8 \\
& y(x)=8 e^{-x} \text { when } x=1.8 \\
& y(x)=24 e^{-x} \text { when } x=2.2
\end{aligned}
$$

Solve for the x value that satisfies the given condition, express your answer to 3 decimal places.

$$
\begin{aligned}
& 6=12 e^{-x} \\
& 1=8 e^{-x} \\
& 12=24 e^{-x}
\end{aligned}
$$

Plot these functions on the same graph for $\mathrm{x}=0$ to 5 .

$$
\begin{aligned}
& y(x)=12 e^{-x} \\
& y(x)=8 e^{-x} \\
& y(x)=24 e^{-x}
\end{aligned}
$$

Part 3: $y(x)=1-e^{-x}(18: 20$ to $23: 30)$
Evaluate the function $y(x)=1-e^{-x}$ at $x=0,1,2,3,4,5$, and plot this function on a graph.
Determine the $x$ that satisfies the condition: $0.6=1-e^{-x}$
Evaluate the function $\mathrm{y}(\mathrm{x})=1-\mathrm{e}^{-\mathrm{x}}$ at $\mathrm{x}=1.3,2.8$, and 3.5
Determine the x values that satisfy these conditions.

$$
\begin{aligned}
& 0.3=1-e^{-x} \\
& 0.8=1-e^{-x} \\
& 0.9=1-e^{-x}
\end{aligned}
$$

## Part 4: $\mathrm{y}(\mathrm{x})=\mathrm{A}\left(1-\mathrm{e}^{-x}\right)(23: 30$ to $31: 58)$

Evaluate the function $\mathrm{y}(\mathrm{x})=6\left(1-\mathrm{e}^{-\mathrm{x}}\right)$ at $\mathrm{x}=1.1$
Determine the x value that satisfies the condition $4=6\left(1-\mathrm{e}^{-x}\right)$
Evaluate the following expressions at the specified x values:

$$
\begin{aligned}
& y(x)=12\left(1-e^{-x}\right) \text { at } x=0.7 \\
& y(x)=8\left(1-e^{-x}\right) \text { at } x=1.8 \\
& y(x)=24\left(1-e^{-x}\right) \text { at } x=2.3
\end{aligned}
$$

Solve the $x$ values that satisfy these given conditions:

$$
\begin{aligned}
& 10=12\left(1-e^{-x}\right) \\
& 5=8\left(1-e^{-x}\right) \\
& 10=24\left(1-e^{-x}\right)
\end{aligned}
$$

Plot these functions on the same graph for $\mathrm{x}=0$ to 5 .

$$
\begin{aligned}
& y(x)=12\left(1-e^{-x}\right) \\
& y(x)=8\left(1-e^{-x}\right) \\
& y(x)=24\left(1-e^{-x}\right)
\end{aligned}
$$

Compare and contrast the functions $y(x)=A e^{-x}$ and $y(x)=A\left(1-e^{-x}\right)$ and plot them on the same graph. Determine the $y$ and $x$ values when the two functions equal each other.

Part 5: time variant functions $y(t)=A e^{-t / \tau}$ and $y(t)=A\left(1-e^{-t / \tau}\right)(31: 58$ to 49:38)
Evaluate the function $y(t)=6 e^{-t / 5 m s}$ when $t=5 \mathrm{~ms}$
Evaluate the function $y(t)=6\left(1-e^{-t / 5 m s}\right)$ when $t=5 \mathrm{~ms}$
Evaluate the function $y(t)=12 \mathrm{e}^{-t / 3 \mathrm{~ms}}$ when $\mathrm{t}=1 \mathrm{~ms}$ and 4 ms
Evaluate the function $y(t)=24\left(1-e^{-t / 2 m s}\right)$ when $t=500 \mu \mathrm{~s}$ and 1.2 ms
Solve for $t$ values that satisfy these conditions:

$$
\begin{aligned}
& 1.7=6 \mathrm{e}^{-\mathrm{t} / 5 \mathrm{~ms}} \\
& 2=6\left(1-\mathrm{e}^{\mathrm{t} / 5 \mathrm{~ms}}\right)
\end{aligned}
$$

Solve for t values that satisfy these conditions:

$$
\begin{aligned}
& 7.5=12 e^{-t / 3 m s} \\
& 19=24\left(1-e^{-t / 2 m s}\right)
\end{aligned}
$$

Plot the functions $\mathrm{y}(\mathrm{t})=8 \mathrm{e}^{-\mathrm{t} / \tau}$ and $\mathrm{y}(\mathrm{t})=8\left(1-\mathrm{e}^{-\mathrm{t} / \tau}\right)$ when $\tau=500 \mu \mathrm{~s}, 250 \mu \mathrm{~s}$, and 1 ms . Discuss the influence of the time constant $\tau$.

Plot the function $y_{1}(t)=24 e^{-t / \tau}$ and $y_{2}(t)=24\left(1-e^{-t / \tau}\right)$ when $\tau=2 \mathrm{~ms}$.
Plot the function $y_{3}(t)=24 e^{-t / \tau}$ and $y_{4}(t)=24\left(1-e^{-t / \tau}\right)$ when $\tau=4 \mathrm{~ms}$.
Discuss the influence of the time constant.
Given these plots, identify which set, top or bottom, has the smaller time constant.


List the 3 properties necessary to conduct expedited analysis of exponential functions. Draw general purpose plots for the purposes of expedited analysis.

## Part 5: Initial values (49:38 to END)

List the formula for a comprehensive exponential function accounting for initial conditions.

Express an exponential transition from 0 to 16 over a span of 25 ms as a time variant formula.

Express an exponential transition from 4 to 16 over a span of 25 ms as a time variant formula.

Express an exponential transition from -8 to 16 over a span of 25 ms as a time variant formula.

Plot the three functions above on the same graph. Compare and contrast how an initial condition influences rate of change and the final value obtained.

Solve for the instantaneous value of the function $y(t)=16-12 e^{-t / 5 m s}$ at $t=5 \mathrm{~ms}$ and 6 ms .

Solve for the instantaneous value of the function $y(t)=16-24 e^{-t / 5 m s}$ at $t=5 \mathrm{~ms}$ and 6 ms .

Determine the $t$ value that satisfies the condition: $10=16-12 e^{-t / 5 m s}$

Determine the $t$ value that satisfies the condition: $0=16-24 \mathrm{e}^{-\mathrm{t} / 5 \mathrm{~ms}}$

Express an exponential transition from 2 to 12 over a span of 15 ms as a time variant function. Determine the instantaneous value at $\mathrm{t}=4.5 \mathrm{~ms}$. Determine the time the function reaches 10 .

Express an exponential transition from -5 to 15 over a span of $300 \mu$ s as a time variant function. Determine the instantaneous value at $\mathrm{t}=100 \mu \mathrm{~s}$. Determine the time the function reaches 0.

Discuss the horizontal and vertical influence of different time constants and initial values when plotting exponential transitions.

