Technical Mathematics

TECHNICAL MATHEMATICS

Morgan Chase

Contents

Introduction

This developmental-level mathematics textbook is intended for career-technical students. It was made possible by a grant from Open Oregon Educational Resources, which supports the development and implementation of high-quality materials at low or no cost to community college and university students.

I hope that this qualifies as a "high quality" textbook, and I hope that it brings a bit of fun to what can often be a boring or intimidating subject. Whether you are a student or instructor, I would love to hear your thoughts on the book and whether it works well for you. Feel free to let me know about any errors you find or suggestions for improvements.

The formatting was optimized for the web version of the textbook, and I know that the pdf versions look rough: strange indenting I can't seem to fix, images that aren't aligned properly, inconvenient page breaks, numbers appearing larger than the surrounding text, etc. I will try to clean up the formatting problems at some point, but it's time for me to make the leap and get this thing out into the world.

Morgan Chase
Clackamas Community College
Oregon City, OR

morganc@clackamas.edu

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Module 1: Order of Operations

To ***evaluate*** an expression means to simplify it and find its value.

Exercises

1. Evaluate by performing the addition first: $12 - 2 + 3$

2. Evaluate by performing the subtraction first: $12 - 2 + 3$

When we evaluate an expression, we want to have a single correct answer. It isn't very helpful for the answer to be "maybe 7, or maybe 13". Mathematicians have decided on an ***order of operations***, which tells us which steps should be done before other steps. Think of them as the rules of the road.

Order of Operations: PEMDAS

P: Work inside of **parentheses** or grouping symbols, following the order PEMDAS as necessary inside the grouping symbols.

E: Evaluate **exponents**.

MD: Perform **multiplications** and **divisions** from left to right.

AS: Perform **additions** and **subtractions** from left to right.

Exercises

Simplify each expression.

3. $12-(2+3)$

4. $12-2+3$

Based on Exercises 3 & 4, we can see that Exercise 1 told us to use the wrong order of operations. If there are no parentheses, we must evaluate $12-2+3$ by *first* performing the subtraction and *then* performing the addition.

Before we move on, you should be aware that there are a handful of ways to show multiplication. All of the following represent 3×4:

$$3\cdot 4 \qquad 3*4 \qquad 3(4) \qquad (3)4 \qquad (3)(4)$$

In this textbook, you will most often see the dot, like $3\cdot 4$, or parentheses directly next to a number, like $3(4)$. We tend to avoid using the 3×4 symbol because it can be mistaken for the letter x.

Exercises

Simplify each expression.

5. $12 \div (3 \cdot 2)$

6. $12 \div 3 \cdot 2$

7. $5(1+3)-2$

8. $5(1)+(3-2)$

An ***exponent*** indicates repeated multiplication. For example, $6^2 = 6 \cdot 6 = 36$ and $4^3 = 4 \cdot 4 \cdot 4 = 64$. The exponent tells us how many factors of the base are being multiplied together.

Exercises

Simplify each expression.

9. $3^2 + 4^2$

10. $(3+4)^2$

11. $(7+3)(7-5)^3$

12. $7+3(7-5)^3$

In the next set of exercises, the only differences are the parentheses, but every exercise has a different answer.

Exercises

Simplify each expression.

13. $39 - 7 \cdot 2 + 3$

14. $(39-7) \cdot 2 + 3$

15. $39 - (7 \cdot 2 + 3)$

16. $39 - 7 \cdot (2+3)$

17. $(39-7)\cdot(2+3)$

It is possible to have grouping symbols nested within grouping symbols; for example, $7+(5^2-(3(17-12\div 4)+2\cdot 5)\div 4)$.

To make it somewhat easier to match up the pairs of left and right parentheses, we can use square brackets instead: $7+(5^2-[3(17-12\div 4)+2\cdot 5]\div 4)$.

Exercises

Simplify the expression.

18. $7+(5^2-[3(17-12\div 4)+2\cdot 5]\div 4)$

A fraction bar is another grouping symbol; it tells us to perform all of the steps on the top and separately perform all of the steps down below. The final step is to divide the top number by the bottom number.

Exercises

Simplify each expression.

19. $\frac{15-1}{6+1}$

20. $\frac{(7+2)\cdot 4}{18\div(3+3)}$

21. $\frac{5\cdot 4^2}{2}$

22. $\frac{(5\cdot 4)^2}{2}$

23. $\frac{(5-1)^2}{2+6}$

24. $(5-1)^2\div 2+6$

We will look at formulas in a later module, but let's finish by translating from words to a mathematical expression.

Exercises

25. You can find the approximate Fahrenheit temperature by doubling the Celsius temperature and adding 30. If the temperature is 9°C, what is the approximate Fahrenheit temperature? Write an expression and simplify it.

26. You can find the approximate Celsius temperature by subtracting 30 from the Fahrenheit temperature and then dividing by 2. If the temperature is 72°F, what is the approximate Celsius temperature? Write an expression and simplify it.

Module 2: Negative Numbers

Negative numbers are a fact of life, from winter temperatures to our bank accounts. Let's practice evaluating expressions involving negative numbers.

Absolute Value

The ***absolute value*** of a number is its distance from 0. You can think of it as the size of a number without identifying it as positive or negative. Numbers with the same absolute value but different signs, such as 3 and -3, are called ***opposites***. The absolute value of -3 is 3, and the absolute value of 3 is also 3.

We use a pair of straight vertical bars to indicate absolute value; for example, $|-3| = 3$ and $|3| = 3$.

Exercises

Evaluate each expression.

1. $|-5|$

2. $|5|$

Adding Negative Numbers

To add two negative numbers, add their absolute values (i.e., ignore the negative signs) and make the final answer negative.

Exercises

Perform each addition.

3. $-8+(-7)$

4. $-13+(-9)$

To add a positive number and a negative number, we *subtract* the smaller absolute value from the larger. If the positive number has the larger absolute value, the final answer is positive. If the negative number has the larger absolute value, the final answer is negative.

Exercises

Perform each addition.

5. $7+(-3)$

6. $-7+3$

7. $14+(-23)$

8. $-14+23$

9. The temperature at noon on a chilly Monday was -7°F. By the next day at noon, the temperature had risen 25°F. What was the temperature at noon on Tuesday?

If an expression consists of only additions, we can break the rules for order of operations and add the numbers in whatever order we choose.

Exercises

Evaluate each expression using any shortcuts that you notice.

10. $-10+4+(-4)+3+10$

11. $-291+73+(-9)+27$

Subtracting Negative Numbers

The image below shows part of a paystub in which an $ 18 payment needed to be made, but the payroll folks wanted to track the payment in the deductions category. Of course, a positive number in the deductions will subtract money away from the paycheck. Here, though, a deduction of negative 18 dollars has the effect of *adding* 18 dollars to the paycheck. Subtracting a negative amount is equivalent to adding a positive amount.

Deductions after Federal Tax	
Faculty Union Dues	$27.00
Stipend for Part Time Faculty	-$18.00
Workers Compensation Hourly Assessment	$0.13

To subtract two signed numbers, we **add** the first number to the **opposite** of the second number.

Exercises

Perform each subtraction.

12. $5 - 2$

13. $2 - 5$

14. $-2 - 5$

15. $-5 - 2$

16. $2 - (-5)$

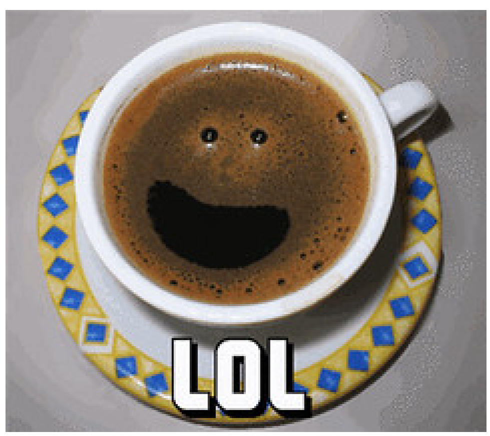
17. $5 - (-2)$

18. $-2 - (-5)$

19. $-5 - (-2)$

20. One day in February, the temperature in Portland, Oregon is 43°F, and the temperature in Portland, Maine is -12°F. What is the difference in temperature?

Multiplying Negative Numbers



Suppose you spend 3 dollars on a coffee every day. We could represent spending 3 dollars as a negative number, -3 dollars. Over the course of a 5-day work week, you would spend 15 dollars, which we could represent as -15 dollars. This shows that $-3 \cdot 5 = -15$, or $5 \cdot -3 = -15$.

If two numbers with **opposite** signs are multiplied, the product is negative.

Exercises

Find each product.

21. $-4 \cdot 3$

22. $5(-8)$

Going back to our coffee example, we saw that $5(-3) = -15$. Therefore, the *opposite* of $5(-3)$ must be positive 15. Because -5 is the opposite of 5, this implies that $-5(-3) = 15$.

If two numbers with the **same** sign are multiplied, the product is positive.

WARNING! These rules are different from the rules for addition; be careful not to mix them up.

Exercises

Find each product.

23. $-2(-9)$

24. $-3(-7)$

Recall that an exponent represents a repeated multiplication. Let's see what happens when we raise a negative number to an exponent.

Exercises

Evaluate each expression.

25. $(-2)^2$

26. $(-2)^3$

27. $(-2)^4$

28. $(-2)^5$

If a negative number is raised to an **odd** power, the result is negative.
If a negative number is raised to an **even** power, the result is positive.

Dividing Negative Numbers

Let's go back to the coffee example we saw earlier: $-3 \cdot 5 = -15$. We can rewrite this fact using division and see that $-15 \div 5 = -3$; a negative divided by a positive gives a negative result. Also, $-15 \div -3 = 5$; a negative divided by a negative gives a positive result. This means that the rules for division work exactly like the rules for multiplication.

If two numbers with **opposite** signs are divided, the quotient is negative.
If two numbers with the **same** sign are divided, the quotient is positive.

Exercises

Find each quotient.

29. $-42 \div 6$

30. $32 \div (-8)$

31. $-27 \div (-3)$

32. $0 \div 4$

33. $0 \div (-4)$

34. $4 \div 0$

Go ahead and check those last three exercises with a calculator. Any surprises?

0 divided by another number is 0.
A number divided by 0 is undefined, or not a real number.

Here's a quick explanation of why $4 \div 0$ can't be a real number. Suppose that there is a mystery number, which we'll call n, such that $4 \div 0 = n$. Then we can rewrite this division as a related multiplication, $n \cdot 0 = 4$. But because 0 times any number is 0, the left side of this equation is 0, and we get the result that $0 = 4$, which doesn't make sense. Therefore, there is no such number n, and $4 \div 0$ cannot be a real number.

Order of Operations with Negative Numbers

P: Work inside of **parentheses** or grouping symbols, following the order PEMDAS as necessary.

E: Evaluate **exponents**.

MD: Perform **multiplications** and **divisions** from left to right.

AS: Perform **additions** and **subtractions** from left to right.

Let's finish up this module with some order of operations practice.

Exercises

Evaluate each expression using the order of operations.

35. $(2-5)^2 \cdot 2 + 1$

36. $2 - 5^2 \cdot (2+1)$

37. $[7(-2)+16] \div 2$

38. $7(-2) + 16 \div 2$

39. $\frac{1-3^4}{2(5)}$

40. $\frac{(1-3)^4}{2} \cdot 5$

Module 3: Decimals

Decimal notation is based on powers of 10: 0.1 is one tenth, 0.01 is one hundredth, 0.001 is one thousandth, and so on.

thousands	hundreds	tens	ones/units	.	tenths	hundredths	thousandths

Exercises

Write each number.

1. ninety and twenty-three hundredths
2. seven and fifty-six thousandths

Adding & Subtracting Decimals

Before you add or subtract decimals, you must line up the decimal points.

Exercises

Add each pair of numbers.

3. $3.75 + 12.8$

4. $71.085 + 112.93$

When subtracting, you may need to add zeros to the first number so you can borrow correctly.

Exercises

Subtract each pair of numbers.

5. $46.57 - 38.29$

6. $82.78 - 67.024$

Multiplying Decimals

To multiply decimal numbers:

1. Temporarily ignore the decimal points.
2. Multiply the numbers as though they are whole numbers.
3. Add the total number of decimal digits in the two numbers you multiplied. The result will have that number of digits to the right of the decimal point.

Note: You do NOT need to line up the decimal points when you are multiplying.

Exercises

Multiply each pair of numbers.

7. $13.5 \cdot 2.9$

8. $4.18 \cdot 3.7$

9. Evie worked 37.5 hours at a pay rate of \$ 17.50 per hour. How much did she earn in total?

Dividing Decimals

Let's review everyone's favorite topic, long division. The three parts of a division are named as follows: dividend ÷ divisor = quotient. When this is written with a long division symbol, the dividend is inside the symbol, the divisor is on the left, and the quotient is the answer we create on top.

$$\begin{array}{r} \text{quotient} \\ \text{divisor}\overline{)\text{dividend}} \end{array}$$

To divide by a decimal:

1. Write in long division form.
2. Move the decimal point of the divisor until it is a whole number.
3. Move the decimal point of the dividend the same number of places to the right.
4. Place the decimal point in the quotient directly above the decimal point in the dividend. Divide the numbers as though they are whole numbers.
5. If necessary, add zeros to the right of the last digit of the dividend to continue.

Exercises

Divide each pair of numbers.

10. $97.4 \div 0.4$

11. $9.74 \div 0.04$

Rounding Numbers

It is often necessary to round a number to a specified place value. We will discuss this in much more depth in a future module, but let's practice rounding now.

Rounding a number:

1. Locate the **rounding digit** in the place to which you are rounding.
2. Look at the **test digit** directly to the right of the rounding digit.
3. If the test digit is 5 or greater, increase the rounding digit by 1 and drop all digits to its right. If the test digit is less than 5, keep the rounding digit the same and drop all digits to its right.

Exercises

Round each number to the indicated place value.

12. $6,473$ (thousands)

13. $6,473$ (hundreds)

14. $6,473$ (tens)

15. 0.7049 (tenths)

16. 0.7049 (hundredths)

17. 0.7049 (thousandths)

If a decimal answer goes on and on, it may be practical to round it off.

Exercises

18. Jerry drove 257 miles using 11 gallons of gas. How many miles per gallon did his car get? Round your result to the nearest tenth.

Module 4: Fractions

5 out of 4 people have trouble with fractions.

A fraction describes equal parts of a whole: $\frac{\text{part}}{\text{whole}}$
Using official math vocabulary: $\frac{\text{numerator}}{\text{denominator}}$

Exercises

The month of April had 11 rainy days and 19 days that were not rainy.
1. What fraction of the days were rainy?
2. What fraction of the days were not rainy?

Simplifying Fractions

Two fractions are ***equivalent*** if they represent the same number. (The same portion of a whole.) To build an equivalent fraction, multiply the numerator and denominator by the same number.

Exercises

3. Write $\frac{4}{5}$ as an equivalent fraction with a denominator of 15.

4. Write $\frac{2}{3}$ as an equivalent fraction with a denominator of 12.

Many fractions can be ***simplified***, or reduced. Here are four special cases.

Exercises

Simplify each fraction, if possible.

5. $\frac{7}{1}$

6. $\frac{7}{7}$

7. $\frac{0}{7}$

8. $\frac{7}{0}$

A fraction is ***completely reduced***, or in ***simplest form***, or in ***lowest terms***, when the numerator and denominator have no common factors other than 1. To reduce a fraction, divide the numerator and denominator by the same number.

Exercises

Reduce each fraction to simplest form.

9. $\frac{9}{12}$

10. $\frac{10}{6}$

Multiplying Fractions

To multiply fractions, multiply the numerators and multiply the denominators straight across. If possible, simplify your answer.

Exercises

Multiply each pair of numbers. Be sure that each answer is in simplest form.

11. $8 \cdot \frac{1}{4}$

12. $\frac{6}{7} \cdot \frac{7}{12}$

13. $\frac{5}{8} \cdot \frac{2}{3}$

14. $\frac{6}{5} \cdot \frac{10}{12}$

To find a fraction **of** a number, multiply.

Exercises

15. To pass his workplace training, Nathan must correctly answer at least $\frac{9}{10}$ of 50 questions. How many questions must he answer correctly to pass the training?

Dividing Fractions

To divide by a fraction, multiply by the reciprocal of the second number. (Flip the second fraction upside-down.)

Exercises

Divide. Be sure that each answer is in simplest form.

16. $12 \div \frac{3}{4}$

17. $\frac{3}{10} \div \frac{1}{2}$

18. Suppose you need to measure 2 cups of flour, but the only scoop you can find is $\frac{1}{3}$ cup. How many scoops of flour will you need?

Comparing Fractions

If two fractions have the same denominator, we can simply compare their numerators.
If two fractions have different denominators, we can rewrite them with a

common denominator and then compare their numerators.

Exercises

19. Cookie recipe A requires $\frac{3}{4}$ cup of sugar, whereas cookie recipe B requires $\frac{2}{3}$ cup of sugar. Which recipe requires more sugar?

Adding & Subtracting Fractions

To add or subtract two fractions with the same denominator, add or subtract the numerators and keep the common denominator.

Exercises

20. Jack ate $\frac{3}{8}$ of a pizza. Mack ate $\frac{1}{8}$ of the pizza. What fraction of the pizza did they eat together?

21. Tracy ate $\frac{5}{6}$ of a pizza. Stacy ate $\frac{1}{6}$ of the pizza. How much more of the pizza did Tracy eat?

To add or subtract two fractions with different denominators, first write them with a common denominator. Then add or subtract them.

Exercises

A $\frac{3}{8}$-inch thick sheet of plywood is going to be laid onto a $\frac{1}{4}$-inch thick sheet of plywood.

22. What is the combined thickness of the two sheets?

23. What is the difference in thickness of the two sheets of plywood?

Jacqueline budgets $\frac{1}{4}$ of her monthly income for food and $\frac{1}{3}$ of her monthly income for rent.

24. What fraction of her monthly income does she budget for these two expenses combined?

25. What fraction more of her monthly income does she budget for her rent than for her food?

Fractions and Decimals

To write a fraction as a decimal, divide the numerator by the denominator.

A decimal that ends (eventually has a remainder of 0) is called a terminating decimal. Fun fact: If the denominator of a fraction has no prime factors other than 2's and 5's, the decimal will terminate. Also, the fraction can be built up to have a denominator of 10, or 100, or $1,000$...

Exercises

Write each fraction as a decimal.

26. $\frac{11}{4}$

27. $\frac{7}{20}$

A decimal that continues a pattern of digits is called a repeating decimal. We can represent the repeating digits by using either an overbar or ellipsis (three dots)...

Exercises

Write each fraction as a decimal.

28. $\frac{5}{9}$

29. $\frac{18}{11}$

Mixed Numbers

A mixed number represents a **sum**. For example, $6\frac{2}{3}$ means $6 + \frac{2}{3}$.

To write a mixed number as an improper fraction:

1. Multiply the whole number part by the denominator.
2. Add this result to the original numerator to get the new numerator.
3. Keep the same denominator.

Exercises

Rewrite each mixed number as an improper fraction.

30. $2\frac{1}{5}$

31. $6\frac{2}{3}$

To write an improper fraction as a mixed number:

1. Divide the numerator by the denominator to get the whole number part.
2. The remainder after dividing is the new numerator.
3. Keep the same denominator.

Exercises

Rewrite each improper fraction as a mixed number.

32. $\frac{23}{2}$

33. $\frac{14}{3}$

Adding or subtracting mixed numbers can be fairly simple or more complicated, depending on the numbers. If adding two mixed numbers would give you an improper fraction as part of your result, you'll need to carry; if subtracting two mixed numbers would give you a negative fraction as part of your result, you'll need to borrow.

Exercises

34. Add: $7\frac{1}{3} + 2\frac{3}{4}$

35. Subtract: $7\frac{1}{3} - 2\frac{3}{4}$

Multiplying or dividing mixed numbers is tricky. Change any mixed numbers into improper fractions before doing the calculation, then change the answer back to a mixed number if possible.

Exercises

36. Multiply: $3\frac{1}{2} \cdot 2\frac{1}{3}$

37. $5\frac{1}{2}$ cups of water will be divided equally into 3 jars. How much water will go into each jar?

Module 5: Accuracy and Significant Figures


In the first few modules, we rarely concerned ourselves with rounding; we assumed that every number we were told was exact and we didn't have to worry about any measurement error. However, every measurement contains some error. A standard sheet of paper is 8.5 inches wide and 11 inches high, but it's possible that the actual measurements could be closer to 8.4999 and 11.0001 inches. Even if we measure something very carefully, with very sensitive instruments, we should assume that there could be some small measurement error.

Exact Values and Approximations

A number is an ***exact value*** if it is the result of counting or a definition.

A number is an ***approximation*** if it is the result of a measurement or of rounding.

Exercises

Identify each number as an exact value or an approximation.

1. An inch is $\frac{1}{12}$ of a foot.

2. This board is 78 inches long.

3. There are 14 students in class.

4. A car's tachometer reads $3,000$ rpm.

5. A right angle measures $90°$.

6. The angle of elevation of a ramp is $4°$.

Suppose a co-worker texts you that they will arrive in 20 minutes. It's hard to tell how precise this number is, because we often round off to the nearest 5 or 10 minutes. You might reasonably expect them to arrive anytime within the next 15 to 25 minutes. If your co-worker texts that they will arrive in 17 minutes, though, it is likely that their GPS told them that more precise number, and you could reasonably expect them to arrive within 16 to 18 minutes.

ACCURACY AND SIGNIFICANT FIGURES

African-American women were vital to NASA's success in the 1960s, as shown in the movie Hidden Figures.

Because measurements are inexact, we need to consider how accurate they are. This requires us to think about ***significant figures***—often abbreviated "sig figs" in conversation—which are the digits in the measurement that we trust to be correct. The ***accuracy*** of a number is equal to the number of significant figures.[1] The following rules aren't particularly difficult to understand but they can take time to absorb and internalize, so we'll include lots of examples and exercises.

Significant Figures

1. All nonzero digits are significant.
   Ex: $12,345$ has five sig figs, and 123.45 has five sig figs.
2. All zeros between other nonzero digits are significant.
   Ex: $10,045$ has five sig figs, and 100.45 has five sig figs.
3. Any zeros to the right of a decimal number are significant.
   Ex: 123 has three sig figs, but 123.00 has five sig figs.
4. Zeros on the left of a decimal number are NOT significant.
   Ex: 0.123 has three sig figs, and 0.00123 has three sig figs.
5. Zeros on the right of a whole number are NOT significant unless they are marked with an overbar.
   Ex: $12,300$ has three sig figs, but $12,30\overline{0}$ has five sig figs.

Another way to think about #4 and #5 above is that zeros that are merely showing the place value—where the decimal point belongs—are NOT significant.

Exercises

Determine the accuracy (i.e., the number of significant figures) of each number.

7. $63,400$

8. $63,040$

9. $63,004$

10. 0.085

11. 0.0805

12. 0.08050

As mentioned above, we use an ***overbar*** to indicate when a zero that looks insignificant is actually significant.
For example, the precision[2] of $7,400$ is the hundreds place; if we rounded anything from $7,350$ to $7,449$ to the nearest hundred, we would write the result as $7,400$. An overbar shows that the number is more precise than it appears. If we rounded anything from $7,395$ to $7,404$ to the nearest ten, the result would be $7,400$, but it isn't clear anymore that the number was rounded to the tens place. Therefore, to show the level of precision, we write the result as $7,4\overline{0}0$. If we rounded anything from $7,399.5$ to $7,400.4$ to the nearest one, the result would be again $7,400$, and we again can't see how precise the rounded number really is. Therefore, to show that the number is precise to the ones place, we write the result as $7,40\overline{0}$.

Exercises

Determine the accuracy (i.e., the number of significant figures) of each number.

13. $8,000$

14. $8,\overline{0}00$

15. $8,0\overline{0}0$

16. $8,00\overline{0}$

Two things to remember: we don't put an overbar over a nonzero digit, and we don't need an overbar for any zeros on the right of a decimal number because those are already understood to be significant.

Accuracy-Based Rounding

As we saw in a previous module about decimals, it is often necessary to round a number. We often round to a certain place value, such as the nearest hundredth, but there is another way to round. ***Accuracy-based rounding*** considers the number of significant figures rather than the place value.

Accuracy-based rounding:

1. Locate the **rounding digit** to which you are rounding by counting from the left until you have the correct number of significant figures.
2. Look at the **test digit** directly to the right of the rounding digit.
3. If the test digit is 5 or greater, increase the rounding digit by 1 and drop all digits to its right. If the test digit is less than 5, keep the rounding digit the same and drop all digits to its right.

Exercises

Round each number so that it has the indicated number of significant figures.

17. $21,837$ (two sig figs)

18. $21,837$ (three sig figs)

19. $21,837$ (four sig figs)

20. 4.2782 (two sig figs)

21. 4.2782 (three sig figs)

22. 4.2782 (four sig figs)

When the rounding digit of a whole number is a 9 that gets rounded up to a 0, we must write an overbar above that 0.

Similarly, when the rounding digit of a decimal number is a 9 that gets rounded up to a 0, we must include the 0 in that decimal place.

Exercises

Round each number so that it has the indicated number of significant figures. Be sure to include trailing zeros or an overbar if necessary.

23. $13,997$ (two sig figs)

24. $13,997$ (three sig figs)

25. $13,997$ (four sig figs)

26. 2.5996 (two sig figs)

27. 2.5996 (three sig figs)

28. 2.5996 (four sig figs)

The mountain we know as Mt. Everest is called *Sagarmatha* in Nepal and *Chomolungma* in Tibet. On December 8, 2020, it was jointly announced by Nepal and China that the summit has an elevation of $29,031.69$ ft, replacing the previously accepted elevation of $29,029$ ft.[3]

Mt. Everest, Lohtse, and Nupse in the early morning

29. Round $29,031.69$ ft to two sig figs.

30. Round $29,031.69$ ft to three sig figs.

31. Round $29,031.69$ ft to four sig figs.

32. Round $29,031.69$ ft to five sig figs.

33. Round $29,031.69$ ft to six sig figs.

Accuracy when Multiplying and Dividing

Suppose you needed to square the number $3\frac{1}{3}$. You could rewrite $3\frac{1}{3}$ as the improper fraction $\frac{10}{3}$ and then figure out that $(\frac{10}{3})^2 = \frac{100}{9}$, which equals the repeating decimal $11.111...$

Because most people prefer decimals to fractions, we might instead round $3\frac{1}{3}$ to 3.3 and find that $3.3^2 = 10.89$. However, this is not accurate because $11.111...$ rounded to the nearest hundredth should be 11.11. The answer 10.89 looks very accurate, but it is a false accuracy because there is round-off error involved. If we round our answer 10.89 to the nearest tenth, we would get 10.9, which is still not accurate because $11.111...$ rounded to the nearest tenth should be 11.1. If we round our answer 10.89 to the nearest whole number, we would get 11, which is accurate because $11.111...$ rounded to the nearest whole number is indeed 11. It turns out that **we should be focusing on the number of significant figures**

rather than the place value; because 3.3 has only two sig figs, our answer must be rounded to two sig figs.

Suppose instead that we round $3\frac{1}{3}$ to 3.33 and find that $3.33^2 = 11.0889$. Again, this is not accurate because $11.111...$ rounded to the nearest ten-thousandth should be 11.1111. If we round 11.0889 to the nearest thousandth, we would get 11.089, which is still not accurate because $11.111...$ rounded to the nearest thousandth should be 11.111. If we round 11.0889 to the nearest hundredth, we would get 11.09, which is still not accurate because $11.111...$ rounded to the nearest hundredth should be 11.11. Only when we round to the nearest tenth do we get an accurate result: 11.0889 rounded to the nearest tenth is 11.1, which is accurate because $11.111...$ rounded to the nearest tenth is indeed 11.1. As above, we need to focus on the number of significant figures rather than the place value; because 3.33 has only three sig figs, our answer must be rounded to three sig figs.

When ***multiplying or dividing*** numbers, the answer must be rounded to the same number of significant figures as the ***least*** accurate of the original numbers.

Don't round off the original numbers; do the necessary calculations first, then round the answer as your last step.

Exercises

Use a calculator to multiply or divide as indicated. Then round to the appropriate level of accuracy.

34. $8.75 \cdot 12.25$

35. $355.12 \cdot 1.8$

36. $77.3 \div 5.375$

37. $53.2 \div 4.5$

38. Suppose you are filling a 5-gallon can of gasoline. The gasoline costs $2.579 per gallon, and you estimate that you will buy 5.0 gallons. How much should you expect to spend?

Notes

1. The terms "significant digits" and "significant figures" are used interchangeably.
2. Precision is different from accuracy, as we'll learn in the next module, but it is mentioned here because it can be difficult to explain one without the other.
3. https://www.washingtonpost.com/world/asia_pacific/mount-everest-height-nepal-china/2020/12/08/a7b3ad1e-389a-11eb-aad9-8959227280c4_story.html

Module 6: Precision and GPE

Precision

The ***precision*** of a number is the place value of the rightmost significant figure. For example, 100.45 is precise to the hundredths place, and $3,840$ is precise to the tens place.

Exercises

Identify the precision (i.e., the place value of the rightmost significant figure) of each number.

1. $63,400$
2. $63,040$
3. $63,004$
4. $8,000$
5. $8,\overline{0}00$
6. $8,0\overline{0}0$
7. $8,00\overline{0}$
8. 0.085
9. 0.0805
10. 0.08050

Precision-Based Rounding

In a previous module about decimals, we used ***precision-based rounding*** because we were rounding to a specified place value; for example, rounding to the nearest tenth. Let's practice this with overbars and trailing zeros.

Precision-based rounding:

1. Locate the **rounding digit** in the place to which you are rounding.
2. Look at the **test digit** directly to the right of the rounding digit.
3. If the test digit is 5 or greater, increase the rounding digit by 1 and drop all digits to its right. If the test digit is less than 5, keep the rounding digit the same and drop all digits to its right.

Remember, when the rounding digit of a whole number is a 9 that gets rounded up to a 0, we must write an overbar above that 0.

Also, when the rounding digit of a decimal number is a 9 that gets rounded up to a 0, we must include the 0 in that decimal place.

Exercises

Round each number to the indicated place value. Be sure to include trailing zeros or an overbar if necessary.

11. $13,997$ (thousands)

12. $13,997$ (hundreds)

13. $13,997$ (tens)

14. 0.5996 (tenths)

15. 0.5996 (hundredths)

16. 0.5996 (thousandths)

Precision when Adding and Subtracting

Suppose the attendance at a large event is estimated at $25,000$ people, but then you see 3 people leave. Is the new estimate $24,997$? No, because the original estimate was precise only to the nearest thousand. We can't start with an imprecise number and finish with a more precise number. If we estimated that $1,000$ people had left, then we could revise our attendance estimate to $24,000$ because this estimate maintains the same level of precision as our original estimate.

When ***adding or subtracting*** numbers with different levels of precision, the answer must be rounded to the same precision as the ***least*** precise of the original numbers.

Don't round off the original numbers; do the necessary calculations first, then round the answer as your last step.

Exercises

Add or subtract as indicated. Round to the appropriate level of precision.

17. Find the combined weight of four packages with the following weights: 9.7 lb, 13 lb, 10.5 lb, 6.1 lb.

18. Find the combined weight of four packages with the following weights: 9.7 lb, 13.0 lb, 10.5 lb, 6.1 lb.

19. While purchasing renter's insurance, Chandra estimates the value of her insurable possessions at \$ $10,200$. After selling some items valued at \$ 375, what would be the revised estimate?

20. Chandra knows that she has roughly \$ 840 in her checking account. After using her debit card to make two purchases of \$ 25.95 and \$ 16.38, how much would she have left in her account?

3 If you are multiplying by an exact number, you can consider this a repeated addition. For example, suppose you measure the weight of an object to be 4.37 ounces and you want to know the weight of three of these objects; multiplying 4.37 times 3 is the same as adding $4.37 + 4.37 + 4.37 = 13.11$ ounces. The precision is still to the hundredths place. The issue of significant figures doesn't apply to exact numbers, so it would be wrong to treat 3 as having only one sig fig. (Treat exact numbers like royalty; their precision is perfect and it would be an insult to even question it.)

Greatest Possible Measurement Error (GPE)

Suppose you are weighing a dog with a scale that displays the weight rounded to the nearest pound. If the scale says Sir Barks-A-Lot weighs 23 pounds, he could weigh anywhere from 22.5 pounds to almost 23.5 pounds. The true weight could be as much as 0.5 pounds above or below the measured weight, which we could write as 23 ± 0.5.

Now suppose you are weighing Sir Barks-A-Lot with a scale that displays the weight rounded to the nearest tenth of a pound. If the scale says Sir Barks-A-Lot weighs 23.0 pounds, we now know that he could weigh anywhere from 22.95 pounds to almost 23.05 pounds. The true weight could be as much as 0.05 pounds above or below the measured weight, which we could write as 23.0 ± 0.05.

As we increase the level of precision in our measurement, we decrease the ***greatest possible measurement error*** or ***GPE***. The GPE is always one half the precision; if the precision is to the nearest tenth, 0.1, the GPE is half of one tenth, or five hundredths, 0.05. The GPE will always be a 5 in the place to the right of the place value of the number's precision.

Another way to think about the GPE is that it gives the range of values that would round off to the number in question. Back to weighing Sir Barks-A-Lot: 23 ± 0.5

tells us a lower value and an upper value. $23 - 0.5 = 22.5$ is the lowest weight that would round up to 23. Similarly, $23 + 0.5 = 23.5$ is the highest weight that would round down to 23. Yes, perhaps we should say 23.49 or 23.499, etc., for the upper limit here, but it is easier to just say 23.5 and agree that 23.5 is the upper limit even though it would round up instead of down. Using inequalities, we could represent 23 ± 0.5 as the range of values $22.5 \leq \text{weight} < 23.5$ instead.

When you are asked to identify the GPE, it may help to think "What are the minimum and maximum numbers that would round to the given number?" For example, suppose the attendance at a Portland Thorns match is estimated to be $14,000$ people. This number is precise to the nearest thousand. The minimum number that would round up to $14,000$ would be $13,500$ (because $13,449$ would round down to $13,000$), and the maximum number that would round down to $14,000$ would be just below $14,500$ (because $14,500$ would round up to $15,000$). Because these numbers are each 500 away from $14,000$, the GPE is 500. If the estimate of $14,000$ is correct to the nearest thousand, we know that the actual attendance is within ±500 of $14,000$.

Exercises

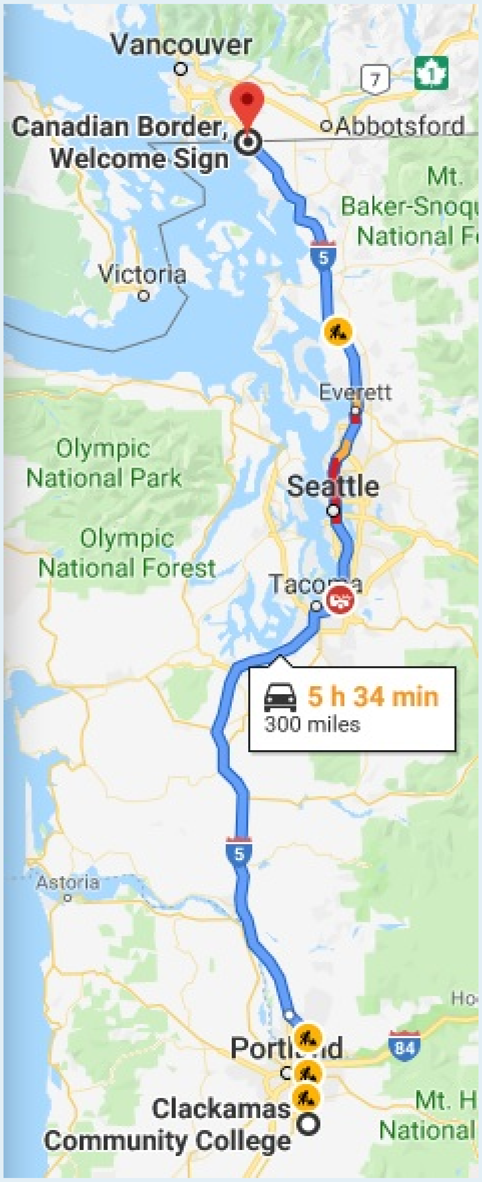
21. A package weighs 3.76 pounds. What is the GPE?

22. A roll of plastic sheeting is 0.00031 inches thick. What is the GPE in inches?

23. Plastic sheeting 0.00031 inches thick is referred to as 0.31 mil. What is the GPE in mils?

Recall from the previous module that the accuracy of a measurement is the number of significant figures. Let's put together the ideas of accuracy, precision, and greatest possible measurement error.

Exercises



Google Maps says that the driving distance from CCC's main campus to the Canadian border is 300 miles. (Note: this is rounded to the nearest mile.)

24. What is the accuracy?

25. What is the precision?

26. What is the GPE?

A new stadium is expected to have around $23,000$ seats.

27. What is the accuracy?

28. What is the precision?

29. What is the GPE?

The capacity of a car's gas tank is 14.2 gallons.

30. What is the accuracy?

31. What is the precision?

32. What is the GPE?

Here is a summary of the important terms from these two modules. It is easy to get them mixed up, but remembering that "precision" and "place value" both start with "p" can be helpful.

Summary of Terms

Significant figures: the digits in a number that we trust to be correct

Accuracy: the number of significant digits

Precision: the place value of the rightmost significant digit

Greatest possible measurement error (GPE): one half the precision

Module 7: Formulas

You may use a calculator throughout this module if needed.

Formulas

A formula is an equation or set of calculations that takes a number (or numbers) as input, and produces an output. The output is often a number, but it could also be a decision such as yes or no. The numbers in a formula are usually represented with letters of the alphabet, which are called ***variables*** because their values can vary. To evaluate a formula, we substitute a number (or numbers) into the formula and then perform the steps using the order of operations.

Note: When a number is written directly next to a variable, it indicates multiplication. For example, $2H$ means $2 \cdot H$.

Exercises

The cost, in dollars, of mailing a large envelope weighing w ounces is calculated by the formula $C = 0.20w + 0.80$.[1]

1. Find the cost of mailing a 3-ounce envelope.

2. Find the cost of mailing a 9-ounce envelope.

Radio Cab charges the following rates for a taxi ride: a fixed fee of \$ 3.50 plus a rate of \$ 2.60 per mile. The total cost, in dollars, of a ride m miles long can be represented by the formula $C = 3.50 + 2.60m$.[2]

3. Find the cost of a 5-mile ride.

4. Find the cost of a 7.5-mile ride.

5. Find the cost of getting in the taxi, then changing your mind and getting out without riding anywhere.

The number of members a state has in the U.S. House of Representatives can be approximated by the formula $R = P \div .7$,[3] where P is the population in millions. The 2010 populations of three states are as follows:[4]

Oregon	3.8 million
Washington	6.7 million
California	37.2 million

Round all answers to the nearest whole number.

6. How many U.S. Representatives does Oregon have?

7. How many U.S. Representatives does Washington have?

8. How many U.S. Representatives does California have?

The number of electoral votes a state has can be approximated by the formula $E = P \div 0.7 + 2$, where P is the population in millions.

9. How many electoral votes does Oregon have?

10. How many electoral votes does Washington have?

11. How many electoral votes does California have?

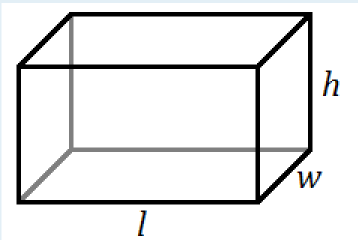
Some formulas require more than one number for the input.

Exercises

When a patient's blood pressure is checked, they are usually told two numbers: the systolic blood pressure (SBP) and the diastolic blood pressure (DBP). The mean arterial pressure (MAP) can be estimated by the following formula: $MAP = \frac{SBP+2\cdot DBP}{3}$. (The units are mm Hg, or millimeters of mercury.) Calculate the mean arterial pressure for each patient.

12. SBP = 120, DBP = 75

13. SBP = 140, DBP = 90



UPS uses this formula to determine the "measurement" of a package with length l, width w, and height h: $m = l + 2w + 2h$.[5] Determine the measurement of a package with the following dimensions.

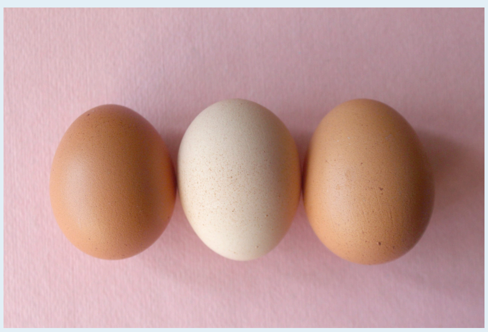
14. length 18 inches, width 12 inches, height 14 inches

15. length 16 inches, width 14 inches, height 15 inches

The next set of exercises involves a formula that gives a yes or no answer.

Exercises

In Australia, a chicken egg is designated "large" if its mass, in grams, satisfies the following formula: $|m - 54.1| \leq 4.1$. Determine whether each egg qualifies as large.[6]



16. Egg 1's mass is 57.8 grams.

17. Egg 2's mass is 58.3 grams.

18. Egg 3's mass is 49.8 grams.

19. Egg 4's mass is 50.0 grams.

Temperature

The Celsius temperature scale is based on the freezing point of water (0°C = 32°F), and the boiling point of water (100°C = 212°F). By subtracting these numbers, we can see that a difference of 180°F is equivalent to 100°C. The ratio $\frac{180}{100}$ reduces to $\frac{9}{5}$, which means that 9 degrees on the Fahrenheit scale is equivalent to 5 degrees on the Celsius scale. (Of course, $\frac{180}{100}$ is also equal to 1.8, which means that 1 degree Celsius is equivalent to 1.8 degrees Fahrenheit.) Because Fahrenheit and Celsius do not have the same zero point, however, we must add or subtract 32 as well. See the formulas below.

Temperature Formulas

$F = \frac{9}{5}C + 32$ or $F = 1.8C + 32$

$C = \frac{5}{9}(F - 32)$ or $C = (F - 32) \div 1.8$

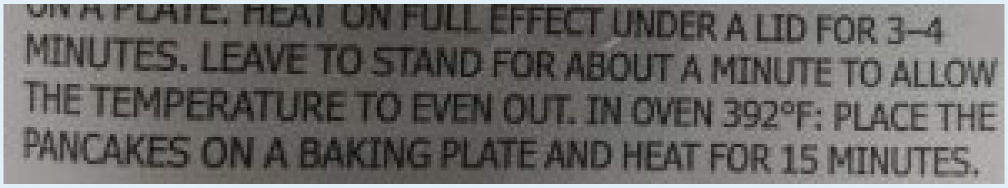
Exercises

20. The temperature on a cool day is 10°C. Convert this temperature to Fahrenheit.

21. Normal body temperature is 98.6°F. What is this temperature in Celsius?

22. The FDA recommends that a freezer be set below -18°C. What is the Fahrenheit equivalent?

23. A package of frozen pancakes from IKEA calls for the oven to be set to 392°F. Clearly, this was originally calculated in Celsius. What is the corresponding Celsius temperature?



Notes

1. https://pe.usps.com/text/dmm300/Notice123.htm#_c037
2. https://www.radiocab.net/services-radio-cab/
3. https://www.pewresearch.org/fact-tank/2018/05/31/u-s-population-keeps-growing-but-house-of-representatives-is-same-size-as-in-taft-era/
4. https://en.wikipedia.org/wiki/List_of_states_and_territories_of_the_United_States_by_population
5. https://www.ups.com/us/en/help-center/packaging-and-supplies/prepare-overize.page
6. https://en.wikipedia.org/wiki/Chicken_egg_sizes

Module 8: Perimeter and Circumference

You may use a calculator throughout this module if needed.

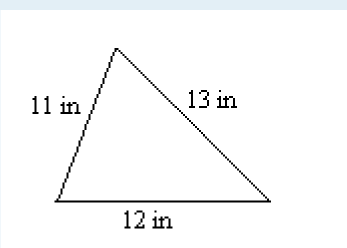
Perimeter

A ***polygon*** is a closed geometric figure with straight sides. Common polygons include triangles, squares, rectangles, parallelograms, trapezoids, pentagons, hexagons, octagons… The ***perimeter*** of a polygon is the distance around the outside. In general, to find the perimeter of a polygon, you can add up the lengths of all of its sides.

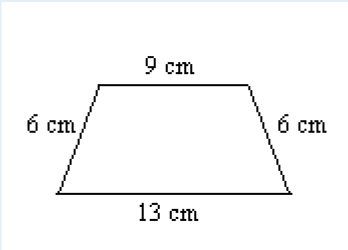
Also, if you haven't already, now is the time to get in the habit of including units in your answers.

Exercises

1. Find the perimeter of the triangle.



2. Find the perimeter of the trapezoid.



If we know that some of the sides of a polygon are equal, we can use a formula as an alternative to adding up all of the lengths individually. The first formula shown below uses the variable s for the side of a square. The rectangle formulas use l for length and w for width, or b for base and h for height; these terms are interchangeable.

Perimeter Formulas

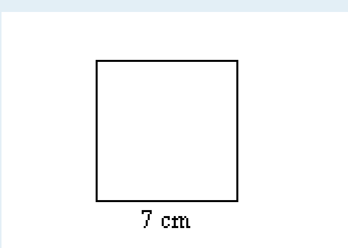
Square: $P = 4s$

Rectangle: $P = 2l + 2w$ or $P = 2b + 2h$

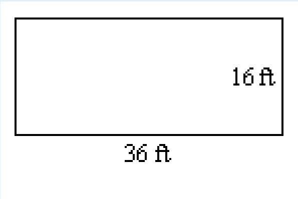
Rectangle: $P = 2(l + w)$ or $P = 2(b + h)$

Exercises

3. Find the perimeter of the square.



4. Find the perimeter of the rectangle.



5. A storage area, which is a rectangle that is 45 feet long and 20 feet wide, needs to be fenced around all four sides. How many feet of fencing is required? (To keep it simple, ignore any gates or other complications.)

6. Giancarlo is putting crown molding around the edge of the ceiling of his living room. If the room is a 12-foot by 16-foot rectangle, how much crown molding does he need?

The sides of a regular polygon are all equal in length. Therefore, multiplying the length of a side by the number of sides will give us the perimeter.

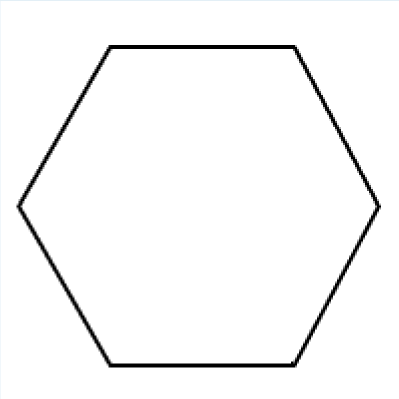
Perimeter Formula

Regular Polygon with n sides of length s: $P = n \cdot s$

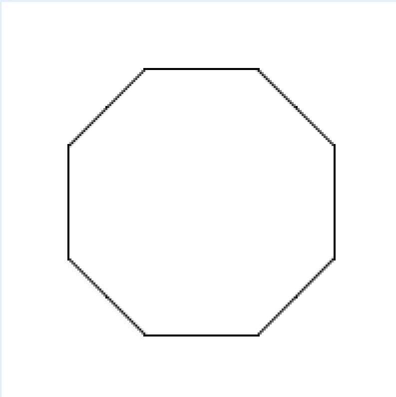
Exercises

Find the perimeter of each regular polygon.

7. Each side of the hexagon is 4 inches long.



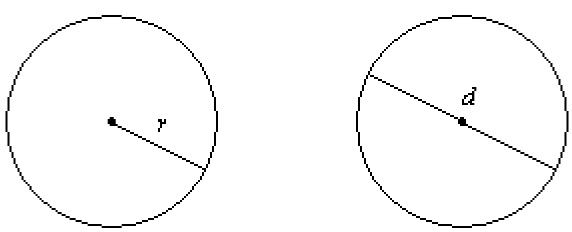
8. Each side of the octagon is 2.5 centimeters long.



Circumference

Instead of calling it the perimeter, the distance around the outside of circle is called the ***circumference***. Let's review some circle vocabulary before moving on.

Every point on a circle is the same distance from its center. This distance from the center to the edge of the circle is called the ***radius***. The distance from one edge to another, through the center of the circle, is called the ***diameter***. As you can see, the diameter is twice the length of the radius.



Throughout history, different civilizations have discovered that the circumference of a circle is slightly more than 3 times the length of its diameter. By the year 2000 BCE, the Babylonians were using the value $3\frac{1}{8} = 3.125$ and the Egyptians were using the value $3\frac{13}{81} \approx 3.1605$.[1] The value $3\frac{1}{7} \approx 3.1429$ is an even better approximation for the ratio of the circumference to the diameter. However, the actual value cannot be written as an exact fraction. It is the irrational number π, pronounced "pie", which is approximately 3.14159.

Circumference Formulas

$C = \pi d$

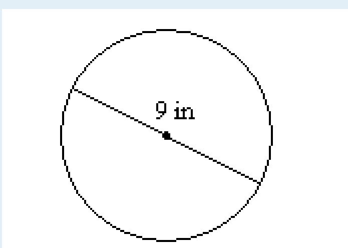
$C = 2\pi r$

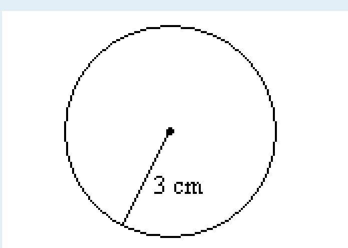
Any scientific calculator will have a π key; using this will give you the most accurate result, although you should be sure to round your answer appropriately. (See this module if you need a refresher on rounding with multiplying or dividing.) Many people use 3.14 as an approximation for π, but this can lead to round-off error; if you must use an approximation, 3.1416 is better than 3.14.

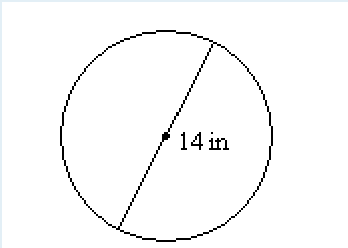
Sometimes we bend the rules in this textbook and ask you to round to a certain place value instead of rounding to a certain number of significant digits.

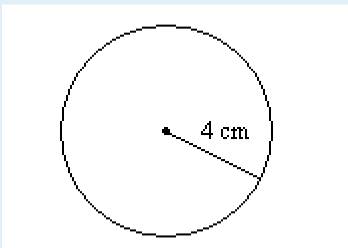
Exercises

Calculate the circumference of each circle. Round to the nearest tenth.

9. 

10. 

11. 

12. 

Notes

1. This information comes from Chapter 1 of the book *A History of Pi* by Petr Beckmann. It is a surprisingly interesting read.

Module 9: Percents Part 1

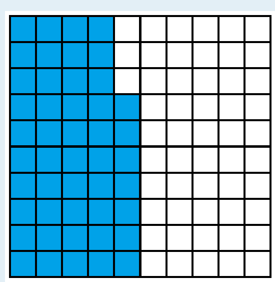
Percent Basics

Percent means "per one hundred". A percent is a ratio or fraction with a denominator of 100.

Exercises

During Super Bowl XLIX between the Seahawks and Patriots, 89 out of 100 television sets in Seattle were tuned to the game.[1]

1. What percent of the television sets were tuned to the game?
2. What percent of the television sets were not tuned to the game?



3. What percent of the squares are shaded?

4. What percent of the squares are not shaded?

To write a percent as a fraction: drop the percent sign, write the number over 100 , and simplify if possible.

Tip: If a percent is greater than 100%, the fraction will be greater than 1. If a percent is less than 1%, the fraction will be less than $\frac{1}{100}$.

Exercises

Write each percent as a fraction, and simplify if possible.

5. About 71% of Earth's surface is covered by water.[2]

6. About 1.3% of Earth's land surface is permanent cropland.[3]

7. About 0.04% of Earth's atmosphere is carbon dioxide.[4]

8. The worldwide number of active Facebook users in the fourth quarter of 2018 was approximately 102% of the number of users in the third quarter of 2018.[5]

To write a percent as a decimal: drop the percent sign and move the decimal point two places to the left.

Exercises

Write each percent from Exercises 5 through 8 as a decimal.

9. 71%

10. 1.3%

11. 0.04%

12. 102%

To write a decimal as a percent: move the decimal point two places to the right and insert a percent sign.

Exercises

Write each decimal number as a percent.

13. 0.23

14. 0.07

15. 0.085

16. 2.5

To write a fraction as a percent, write the fraction as a decimal by dividing the numerator by the denominator, then move the decimal point two places to the right and insert a percent sign.

Alternate method: Recall from the fractions module that if the denominator of a fraction has no prime factors other than 2's and 5's, then the fraction can be built up to have a denominator of 10, or 100, or $1,000$...

Exercises

17. 7 out of 25 students were tardy on Wednesday. Write $\frac{7}{25}$ as a percent.

18. A package of 24 m&m's contained 3 orange m&m's. Write $\frac{3}{24}$ as a percent.

Solving Percent Problems: Finding the Amount

You may use a calculator throughout the remainder of this module.

We often use the words ***amount*** and ***base*** in a percent problem. The ***amount*** is the answer we get after finding the percent of the original number. The ***base*** is the original number, the number we find the percent of. (You may also think of the amount as the part, and the base as the whole.) We can call the percent the ***rate***.

$$\text{Amount} = \text{Rate} \cdot \text{Base}$$

$$A = R \cdot B$$

Be sure to change the percent to a decimal before multiplying.

Exercises

19. What is 9% of 350?

20. 30% of 75 is what number?

21. Find 13.5% of 500.

22. 125% of 80 is equal to what amount?

23. What number is 40% of 96.5?

24. Calculate 0.5% of 450.

Suppose you buy an electric drill with a retail price of $ 109.97 in a city with 8.5% sales tax.

25. Find the amount of the tax. Round to the nearest cent, if necessary.

26. How much do you pay in total?

Notes

1. https://twitter.com/darrenrovell/status/562258101337067521
2. https://en.wikipedia.org/wiki/Earth#Surface
3. https://en.wikipedia.org/wiki/Earth#Surface
4. https://en.wikipedia.org/wiki/Atmosphere_of_Earth#Composition
5. https://www.statista.com/statistics/264810/number-of-monthly-active-facebook-users-worldwide/

Module 10: Ratios, Rates, Proportions

Ratios & Rates

A ***ratio*** is the quotient of two numbers or the quotient of two quantities with the same units.

When writing a ratio as a fraction, the first quantity is the numerator and the second quantity is the denominator.

Exercises

1. Find the ratio of 45 minutes to 2 hours. Simplify the fraction, if possible.

A ***rate*** is the quotient of two quantities with different units. You must include the units.

When writing a rate as a fraction, the first quantity is the numerator and the second quantity is the denominator. Simplify the fraction, if possible. Include the units in the fraction.

Exercises

2. A car travels 105 miles in 2 hours. Write the rate as a fraction.

A ***unit rate*** has a denominator of 1. If necessary, divide the numerator by the denominator and express the rate as a mixed number or decimal.

Exercises

3. A car travels 105 miles in 2 hours. Write as a unit rate.

A ***unit price*** is a rate with the price in the numerator and a denominator equal to 1. The unit price tells the cost of one unit or one item. You can also simply divide the cost by the size or number of items.

Exercises

4. An 18-ounce box of cereal costs \$ 3.59. Find the unit price.

5. A 12-ounce box of cereal costs \$ 2.99. Find the unit price.

6. Which box has a lower unit price?

Proportions

A ***proportion*** says that two ratios (or rates) are equal.

Exercises

Determine whether each proportion is true or false by simplifying each fraction.

7. $\frac{6}{8} = \frac{21}{28}$

8. $\frac{10}{15} = \frac{16}{20}$

A common method of determining whether a proportion is true or false is called ***cross-multiplying*** or finding the ***cross products***. We multiply diagonally across the equal sign. In a true proportion, the cross products are equal.

$$\frac{a}{b} = \frac{c}{d} \rightarrow \ a \cdot d = b \cdot c$$

Exercises

Determine whether each proportion is true or false by cross-multiplying.

9. $\frac{6}{8} = \frac{21}{28}$

10. $\frac{10}{15} = \frac{16}{20}$

11. $\frac{14}{4} = \frac{15}{5}$

12. $\frac{0.8}{4} = \frac{5}{25}$

As we saw in a previous module, we can use a variable to stand for a missing number. If a proportion has a missing number, we can use cross multiplication to solve for the missing number. This is as close to algebra as we get in this textbook.

To solve a proportion for a variable:

1. Set the cross products equal to form an equation of the form $a \cdot d = b \cdot c$.
2. Isolate the variable by rewriting the multiplication equation as a division equation.
3. Check the solution by substituting the answer into the original proportion and finding the cross products.

You may discover slightly different methods that you prefer.[1] If you think "Hey, can't I do this a different way?", you may be correct.

Exercises

Solve for the variable.

13. $\frac{8}{10} = \frac{x}{15}$

14. $\frac{3}{2} = \frac{7.5}{n}$

15. $\frac{3}{k} = \frac{18}{24}$

16. $\frac{w}{6} = \frac{15}{9}$

17. $\frac{5}{4} = \frac{13}{x}$

18. $\frac{3.2}{7.2} = \frac{m}{4.5}$ (calculator recommended)

Problems that involve rates, ratios, scale models, etc. can be solved with proportions. When solving a real-world problem using a proportion, be consistent with the units.

Exercises

19. Tonisha drove her car 320 miles and used 12.5 gallons of gas. At this rate, how far could she drive using 10 gallons of gas?

20. Marcus worked 14 hours and earned \$ 210. At the same rate of pay, how long would he have to work to earn \$ 300?

21. A picture of your author appearing on *Jeopardy!* that is 375 pixels high and 475 pixels wide needs to be reduced in size so that it is 150 pixels high. If the height and width are kept proportional, what is the width of the picture after it has been reduced?

Notes

1. The steps in the box are designed to avoid mentioning the algebraic step of dividing both sides of the equation by a number. If you are comfortable with basic algebra, then you would phrase step 2 differently.

Module 11: Scientific Notation

Powers of Ten

Decimal notation is based on powers of 10: 0.1 is $\frac{1}{10^1}$, 0.01 is $\frac{1}{10^2}$, 0.001 is $\frac{1}{10^3}$, and so on.

We represent these powers with negative exponents: $\frac{1}{10^1} = 10^{-1}$, $\frac{1}{10^2} = 10^{-2}$, $\frac{1}{10^3} = 10^{-3}$, etc.

> Negative exponents: $\frac{1}{10^n} = 10^{-n}$

Note: This is true for any base, not just 10*, but we will focus only on* 10 *in this course.*

With our base 10 number system, any power of 10 can be written as a 1 in a certain decimal place.

10^4	10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}	10^{-4}
$10,000$	$1,000$	100	10	1	0.1	0.01	0.001	0.0001

If you haven't watched the video "Powers of Ten" from 1977 on YouTube, take ten minutes right now and check it out. Your mind will never be the same again.

Scientific Notation

Let's consider how we could rewrite some different numbers using these powers of 10.

Let's take $50,000$ as an example. $50,000$ is equal to $5 \times 10,000$ or 5×10^4.[1]

Looking in the other direction, a decimal such as 0.0007 is equal to 7×0.0001 or 7×10^{-4}.

The idea behind scientific notation is that we can represent very large or very small numbers in a more compact format: a number between 1 and 10, multiplied by a power of 10.

A number is written in scientific notation if it is written in the form $a \times 10^n$, where n is an integer and a is any real number such that $1 \leq a < 10$.

Note: An integer is a number with no fraction or decimal part: ... -3*,* -2*,* -1*,* 0*,* 1*,* 2*,* 3 *...*

Exercises

1. The mass of the Earth is approximately $5,970,000,000,000,000,000,000,000$ kilograms. The mass of Mars is approximately $639,000,000,000,000,000,000,000$ kilograms. Can you determine which mass is larger?

Clearly, it is difficult to keep track of all those zeros. Let's rewrite those huge numbers using scientific notation.

Exercises

2. The mass of the Earth is approximately 5.97×10^{24} kilograms. The mass of Mars is approximately 6.39×10^{23} kilograms. Can you determine which mass is larger?

It is much easier to compare the powers of 10 and determine that the mass of the Earth is larger because it has a larger power of 10. You may be familiar with the term ***order ofmagnitude***; this simply refers to the difference in the powers of 10 of the two numbers. Earth's mass is one order of magnitude larger because 24 is 1 more than 23.

We can apply scientific notation to small decimals as well.

Exercises

3. The radius of a hydrogen atom is approximately 0.000000000053 meters. The radius of a chlorine atom is approximately 0.00000000018 meters. Can you determine which radius is larger?

Again, keeping track of all those zeros is a chore. Let's rewrite those decimal numbers using scientific notation.

Exercises

4. The radius of a hydrogen atom is approximately 5.3×10^{-11} meters. The radius of a chlorine atom is approximately 1.8×10^{-10} meters. Can you determine which radius is larger?

The radius of the chlorine atom is larger because it has a larger power of 10; the digits 1 and 8 for chlorine begin in the tenth decimal place, but the digits 5 and 3 for hydrogen begin in the eleventh decimal place.

Scientific notation is very helpful for really large numbers, like the mass of a planet, or really small numbers, like the radius of an atom. It allows us to do calculations or compare numbers without going cross-eyed counting all those zeros.

Exercises

Write each of the following numbers in scientific notation.

5. $1,234$

6. $10,200,000$

7. 0.00087

8. 0.0732

Convert the following numbers from scientific notation to standard decimal notation.

9. 3.5×10^4

10. 9.012×10^7

11. 8.25×10^{-3}

12. 1.4×10^{-5}

You may be familiar with a shortcut for multiplying numbers with zeros on the end; for example, to multiply $300 \times 4,000$, we can multiply the significant digits $3 \times 4 = 12$ and count up the total number of zeros, which is five, and write five zeros on the back end of the 12: $1,200,000$. This shortcut can be applied to numbers in scientific notation.

To multiply powers of 10, add the exponents: $10^m \cdot 10^n = 10^{m+n}$

Exercises

Multiply each of the following and write the answer in scientific notation.

13. $(2 \times 10^3)(4 \times 10^4)$

14. $(5 \times 10^4)(7 \times 10^8)$

15. $(3 \times 10^{-2})(2 \times 10^{-3})$

16. $(8 \times 10^{-5})(6 \times 10^9)$

When the numbers get messy, it's probably a good idea to use a calculator. If you are dividing numbers in scientific notation with a calculator, you may need to use parentheses carefully.

Exercises

The mass of a proton is 1.67×10^{-27} kg. The mass of an electron is 9.11×10^{-31} kg.

17. Divide these numbers using a calculator to determine approximately how many times greater the mass of a proton is than the mass of an electron.

18. What is the approximate mass of one million protons? (Note: one million is 10^6.)

19. What is the approximate mass of one billion protons? (Note: one billion is 10^9.)

Engineering Notation

Closely related to scientific notation is ***engineering notation***, which uses only multiples of $1,000$. This is the way large numbers are often reported in the news; if roughly $37,000$ people live in Oregon City, we say "thirty-seven thousand" and we might see it written as "37 thousand"; it would be unusual to think of it as $3.7 \times 10,000$ and report the number as "three point seven ten thousands".

One thousand = 10^3, one million = 10^6, one billion = 10^9, one trillion = 10^{12}, and so on.

In engineering notation, the power of 10 is always a multiple of 3, and the other part of the number must be between 1 and $1,000$.

A number is written in engineering notation if it is written in the form $a \times 10^n$, where n is a multiple of 3 and a is any real number such that $1 \leq a < 1,000$.

Note: Prefixes for large numbers such as kilo, mega, giga, and tera are essentially engineering notation, as are prefixes for small numbers such as micro, nano, and pico. We'll see these in another module.

Exercises

Write each number in engineering notation, then in scientific notation.

20. The U.S. population is around 330.2 million people.[2]

21. The world population is around 7.68 billion people.[3]

22. The U.S. national debt is around 26.6 trillion dollars.[4]

Notes

1. For some reason, although we generally try to avoid using the "x" shaped multiplication symbol, it is frequently used with scientific notation.
2. August 27, 2020 estimate from *https://www.census.gov/popclock/*
3. August 27, 2020 estimate from *https://www.census.gov/popclock/*
4. August 27, 2020 data from *https://fiscaldata.treasury.gov/datasets/debt-to-the-penny/debt-to-the-penny*

Module 12: Percents Part 2 and Error Analysis

You may use a calculator throughout this module.

Recall: The ***amount*** is the answer we get after finding the percent of the original number. The ***base*** is the original number, the number we find the percent of. We can call the percent the ***rate***.

When we looked at percents in a previous module, we focused on finding the amount. In this module, we will learn how to find the percentage rate and the base.

$$\text{Amount} = \text{Rate} \cdot \text{Base}$$

$$A = R \cdot B$$

We can translate from words into algebra.

- “is” means equals
- “of” means multiply
- “what” means a variable

Solving Percent Problems: Finding the Rate

Suppose you earned 56 points on a 60-point quiz. To figure out your grade as a percent, you need to answer the question “56 is what percent of 60?” We can translate this sentence into the equation $56 = R \cdot 60$.

Exercises

1. 56 is what percent of 60?

2. What percent of 120 is 45?

Be aware that this method gives us the answer in decimal form and we must move the decimal point to convert the answer to a percent.

Also, if the instructions don’t explicitly tell you how to round your answer, use your best judgment: to the nearest whole percent or nearest tenth of a percent, to two or three significant figures, etc.

Solving Percent Problems: Finding the Base

Suppose you earn 2% cash rewards for the amount you charge on your credit card. If you want to earn $\$\ 50$ in cash rewards, how much do you need to charge on your card? To figure this out, you need to answer the question “50 is 2% of what number?” We can translate this into the equation $50 = 0.02 \cdot B$.

Exercises

3. $\$\ 50$ is 2% of what number?

4. 5% of what number is 36?

Solving Percent Problems: Using Proportions

Recall that a percent is a ratio, a fraction out of 100. Instead of translating word for word as we have just been doing, we can set up a proportion with the percentage rate over 100. Because the base is the original amount, it corresponds to 100%.

$$\frac{\text{amount}}{\text{base}} = \frac{\text{percent}}{100}$$

Let's try Exercises 1 through 4 again, using proportions.

Exercises

5. 56 is what percent of 60?

6. What percent of 120 is 45?

7. \$ 50 is 2% of what number?

8. 5% of what number is 36?

Now that we have looked at both methods, you are free to use whichever method you prefer: percent equations or proportions.

Exercises

9. An 18% tip will be added to a dinner that cost \$ 107.50. What is the amount of the tip?

10. The University of Oregon women's basketball team made 13 of the 29 three-points shots they attempted during a game against UNC. What percent of their three-point shots did the team make?

11. 45% of the people surveyed answered "yes" to a poll question. If 180 people answered "yes", how many people were surveyed altogether?

Solving Percent Problems: Percent Increase

When a quantity changes, it is often useful to know by what percent it changed. If the price of a candy bar is increased by 50 cents, you might be annoyed because it's it's a relatively large percentage of the original price. If the price of a car is increased by 50 cents, though, you wouldn't care because it's such a small percentage of the original price.

To find the percent of increase:

1. Subtract the two numbers to find the amount of increase.
2. Using this result as the amount and the **original** number as the base, find the unknown percent.

Notice that we always use the ***original*** number for the base, the number that occurred earlier in time. In the case of a percent increase, this is the smaller of the two numbers.

Exercises

12. The price of a candy bar increased from \$ 0.89 to \$ 1.39. By what percent did the price increase?

13. The population of Portland in 2010 was $583,793$. The estimated population in 2019 was $654,741$. Find the percent of increase in the population.[1]

Solving Percent Problems: Percent Decrease

Finding the percent decrease in a number is very similar.

To find the percent of decrease:

1. Subtract the two numbers to find the amount of decrease.
2. Using this result as the amount and the **original** number as the base, find the unknown percent.

Again, we always use the ***original*** number for the base, the number that occurred earlier in time. For a percent decrease, this is the larger of the two numbers.

Exercises

14. During a sale, the price of a candy bar was reduced from $ 1.39 to $ 0.89. By what percent did the price decrease?

15. The number of students enrolled at Clackamas Community College decreased from $7,439$ in Summer 2019 to $4,781$ in Summer 2020. Find the percent of decrease in enrollment.

Relative Error

In an earlier module, we said that a measurement will always include some error, no matter how carefully we measure. It can be helpful to consider the size of the error relative to the size of what is being measured. As we saw in the examples above, a difference of 50 cents is important when we're pricing candy bars but insignificant when we're pricing cars. In the same way, an error of an eighth of an inch could be a deal-breaker when you're trying to fit a screen into a window frame, but an eighth of an inch is insignificant when you're measuring the length of your garage.

The ***expected outcome*** is what the number would be in a perfect world. If a window screen is supposed to be exactly 25 inches wide, we call this the expected outcome, and we treat it as though it has infinitely many significant digits. In theory, the expected outcome is $25.000000...$

To find the ***absolute error***, we subtract the measurement and the expected outcome. Because we always treat the expected outcome as though it has unlimited significant figures, the absolute error should have the same precision (place value) as the *measurement*, not the expected outcome.

To find the ***relative error***, we divide the absolute error by the expected outcome. We usually express the relative error as a percent. In fact, the procedure for finding the relative error is identical to the procedures for finding a percent increase or percent decrease!

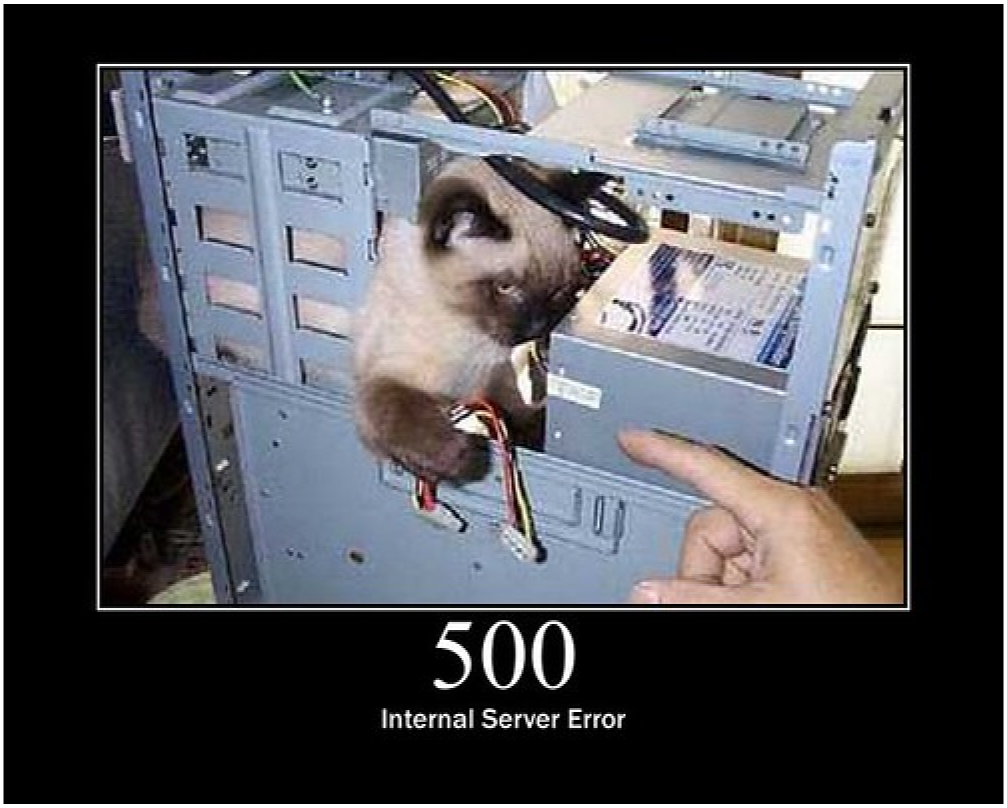
To find the relative error:

1. Subtract the two numbers to find the absolute error.
2. Using the **absolute error** as the amount and the **expected outcome** as the base, find the unknown percent.

Exercises

16. A window screen is measured to be $25\frac{3}{16}$ inches wide instead of the advertised 25 inches. Determine the relative error, rounded to the nearest tenth of a percent.

17. The contents of a box of cereal are supposed to weigh 10.8 ounces, but they are measured at 10.67 ounces. Determine the relative error, rounded to the nearest tenth of a percent.



Tolerance

The ***tolerance*** is the maximum amount that a measurement is allowed to differ from the expected outcome. For example, the U.S. Mint needs its coins to have a consistent size and weight so that they will work in vending machines. A dime (10 cents) weighs 2.268 grams, with a tolerance of ± 0.091 grams.[2] This tells us that the minimum acceptable weight is $2.268 - 0.091 = 2.177$ grams, and the maximum acceptable weight is $2.268 + 0.091 = 2.359$ grams. A dime with a weight outside of the range $2.177 \leq \text{weight} \leq 2.359$ would be unacceptable.

Exercises

A U.S. nickel (5 cents) weighs 5.000 grams with a tolerance of ± 0.194 grams.

18. Determine the lowest acceptable weight and highest acceptable weight of a nickel.

19. Determine the relative error of a nickel that weighs 5.21 grams.

A U.S. quarter (25 cents) weighs 5.670 grams with a tolerance of ± 0.227 grams.

20. Determine the lowest acceptable weight and highest acceptable weight of a quarter.

21. Determine the relative error of a quarter that weighs 5.43 grams.

Notes

1. https://www.census.gov/quickfacts/fact/table/portlandcityoregon,OR,US/PST045219
2. https://www.usmint.gov/learn/coin-and-medal-programs/coin-specifications and https://www.thesprucecrafts.com/how-much-do-coins-weigh-4171330

Module 13: The US Measurement System

You may use a calculator throughout this module.

Robin the cat has the spirit of '76.

This system used to be called the English system, but now the U.S. has the dubious honor of being associated with the system that uses inches, feet, miles, ounces, pounds, cups, gallons, etc. To convert from one unit to another, we often have to perform messy calculations like dividing by 16 or multiplying by $5,280$.

We could solve these unit conversions using proportions, but there is another method than is more versatile, especially when a conversion requires more than one step. This method goes by various names, such as *dimensional analysis* or the *factor label method*. The basic idea is to begin with the measurement you know, then multiply it by a conversion ratio that will cancel the units you don't want and replace it with the units you do want.

It's okay if you don't have the conversion ratios memorized; just be sure to have them available. If you discover other conversion ratios that aren't provided here, go ahead and write them down!

U.S. System: Measurements of Length

1 foot = 12 inches

1 yard = 3 feet

1 mile = $5,280$ feet

Exercises

1. How many inches are in 4.5 feet?

2. How many feet make up 18 yards?

3. 1 yard is equal to how many inches?

4. 1 mile is equivalent to how many yards?

5. How many feet is 176 inches?

6. 45 feet is what length in yards?

7. Convert $10,560$ feet into miles.

8. How many yards are the same as $1,080$ inches?

Notice that Exercises 3 & 4 give us two more conversion ratios that we could add to our list.

U.S. System: Measurements of Weight or Mass

1 pound = 16 ounces

1 ton = $2,000$ pounds

Exercises

9. How many ounces are in 2.5 pounds?

10. How many pounds are equivalent to 1.2 tons?

11. Convert 300 ounces to pounds.

12. 1 ton is equivalent to what number of ounces?

U.S. System: Measurements of Volume or Capacity

1 cup = 8 fluid ounces

1 pint = 2 cups

1 quart = 2 pints

1 gallon = 4 quarts

There are plenty of other conversions that could be provided, such as the number of fluid ounces in a gallon, but let's keep the list relatively short.

Exercises

13. How many fluid ounces are in 6 cups?

14. How many pints are in 3.5 quarts?

15. 1 gallon is equal to how many pints?

16. How many cups equal 1.25 quarts?

17. Convert 20 cups into gallons.

18. How many fluid ounces are in one half gallon?

U.S. System: Using Mixed Units of Measurement

Measurements are frequently given with mixed units, such as a person's height being given as 5 ft 7 in instead of 67 in, or a newborn baby's weight being given as 8 lb 3 oz instead of 131 oz. This can sometimes make the calculations more complicated, but if you can convert between improper fractions and mixed numbers, you can handle this.

Exercises

19. A bag of apples weighs 55 ounces. What is its weight in pounds and ounces?

20. A carton of orange juice contains 59 fluid ounces. Determine its volume in cups and fluid ounces.

21. A hallway is 182 inches long. Give its length in feet and inches.

22. The maximum loaded weight of a Ford F-150 pickup truck is $8,500$ lb. Convert this weight into tons and pounds.

We'll finish up this module by adding and subtracting with mixed units. Again, it may help to think of them as mixed numbers, with a whole number part and a fractional part.

Exercises

Comet weighs 8 lb 7 oz and Fred weighs 11 lb 9 oz.

23. Comet and Fred are being put into a cat carrier together. What is their combined weight?

24. How much heavier is Fred than Comet?

Two tables are 5 ft 3 in long and 3 ft 10 in long.

25. If the two tables are placed end to end, what is their combined length?

26. What is the difference in length between the two tables?

Module 14: The Metric System

You will NOT need a calculator for this module.

The metric system was first implemented following the French Revolution; if we're overthrowing the monarchy, why should we use a unit of a "foot" that is based on the length of a king's foot?

The metric system was designed to be based on the natural world, and different units are related to each other by powers of 10 instead of weird numbers like 3, 12, 16, and $5,280$... This makes converting between metric units incredibly simple.

kilo- (k)	hecta- (h)	deka- (da)	**[base unit]**	deci- (d)	centi- (c)	milli- (m)
$1,000$	100	10	1	0.1	0.01	0.001
$1,000$	100	10	1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1{,}000}$
10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}

Notice that because deka- and deci- both start with d, the abbreviation for deka- is da.

Metric System: Measurements of Length

The base unit of length is the ***meter***, which is a bit longer than a yard (three feet). Because the prefix ***kilo-*** means one thousand, 1 kilometer is $1,000$ meters. (One kilometer is around six tenths of a mile.) Similarly, because the prefix ***centi-*** means one hundredth, 1 centimeter is $\frac{1}{100}$ of a meter, or 1 meter is 100 centimeters. (One centimeter is roughly the thickness of a pen.) And because the prefix ***milli-*** means one thousandth, 1 millimeter is $\frac{1}{1{,}000}$ of a meter, or 1 meter is $1,000$ millimeters. (One millimeter is roughly the thickness of a credit card.)

Exercises

From each of the four choices, choose the most reasonable measure.

1. The length of a car:
5 kilometers, 5 meters, 5 centimeters, 5 millimeters

2. The height of a notebook:
28 kilometers, 28 meters, 28 centimeters, 28 millimeters

3. The distance to the next town:
3.8 kilometers, 3.8 meters, 3.8 centimeters, 3.8 millimeters

4. An adult woman's height:
1.6 kilometers, 1.6 meters, 1.6 centimeters, 1.6 millimeters

5. An adult woman's height:
160 kilometers, 160 meters, 160 centimeters, 160 millimeters

6. The thickness of a pane of glass:
3 kilometers, 3 meters, 3 centimeters, 3 millimeters

kilo- (km)	hecta- (hm)	deka- (dam)	**meter (m)**	deci- (dm)	centi- (cm)	milli- (mm)
$1,000$	100	10	1	0.1	0.01	0.001
$1,000$	100	10	1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1{,}000}$
10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}

To convert metric units, you can simply move the decimal point left or right the number of places indicated in the table above. No calculator required!

Exercises

7. Convert 3.7 meters to centimeters.

8. Convert 3.7 meters to millimeters.

9. Convert 2.45 kilometers to meters.

10. Convert 2.45 kilometers to centimeters.

11. Convert 342 millimeters to meters.

12. Convert 342 millimeters to centimeters.

13. Convert 528 meters to kilometers.

14. Convert 45 centimeters to meters.

Metric System: Measurements of Weight or Mass

The base unit for mass is the ***gram***, which is approximately the mass of a paper clip. A ***kilogram*** is $1,000$ grams; as we'll see in the next module, this is around 2.2 pounds. The active ingredients in medicines may be measured using the ***milligram***, or possibly the ***microgram***, which we will come back to in a future module. For now, we will focus on the prefixes between kilo- and milli-.

Exercises

From each of the three choices, choose the most reasonable measure.

15. The mass of an apple:
100 kilograms, 100 grams, 100 milligrams

16. The mass of an adult man:
80 kilograms, 80 grams, 80 milligrams

17. The amount of active ingredient in a pain relief pill:
500 kilograms, 500 grams, 500 milligrams

18. The base vehicle weight of a GMC Sierra 1500:
$2,000$ kilograms, $2,000$ grams, $2,000$ milligrams

kilo- (kg)	hecta- (hg)	deka- (dag)	**gram (g)**	deci- (dg)	centi- (cg)	milli- (mg)
$1,000$	100	10	1	0.1	0.01	0.001
$1,000$	100	10	1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1,000}$
10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}

This table is identical to the previous table; the only difference is that the base unit "meter" has been replaced by "gram". This means that converting metric units of mass is exactly the same process as converting metric units of length; just move the decimal point!

Exercises

19. Convert 0.813 grams to centigrams.

20. Convert 0.813 grams to milligrams.

21. Convert 1.25 kilograms to grams.

22. Convert 1.25 kilograms to milligrams.

23. Convert 960 milligrams to grams.

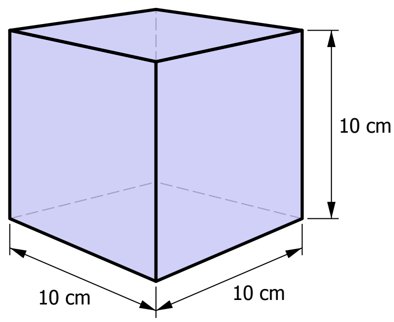
24. Convert 960 milligrams to centigrams.

25. Convert $1,350$ grams to dekagrams.

26. Convert 7.5 centigrams to grams.

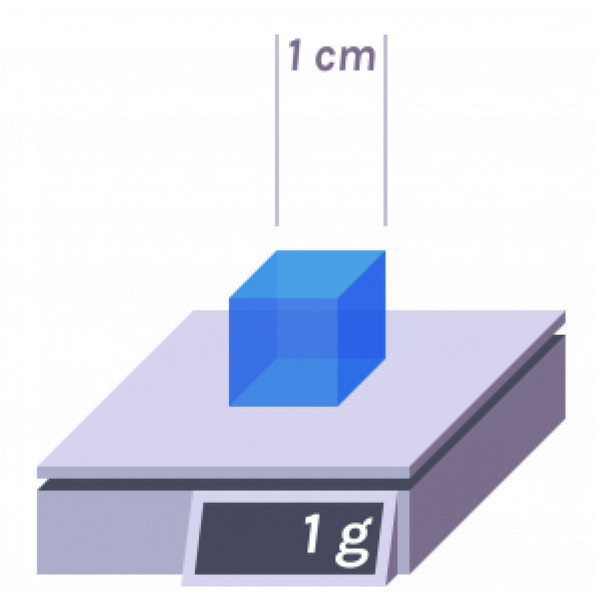
Metric System: Measurements of Volume or Capacity

The base unit of volume is the ***liter***, which is slightly larger than one quart. The ***milliliter*** is also commonly used; of course, there are $1,000$ milliliters in 1 liter.



1 liter is equivalent to a cube with sides of 10 centimeters.

In case you were wondering, the units of volume, length, and mass are all connected; one cubic centimeter (a cube with each side equal to 1 cm) has the same volume as one milliliter, and one milliliter of water has a mass of one gram.



Exercises

From each of the two choices, choose the more reasonable measure.

27. The capacity of a car's gas tank: 50 liters, 50 milliliters

28. A dosage of liquid cough medicine: 30 liters, 30 milliliters

kilo- (kL)	hecta- (hL)	deka- (daL)	**liter (L)**	deci- (dL)	centi- (cL)	milli- (mL)
$1,000$	100	10	1	0.1	0.01	0.001
$1,000$	100	10	1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1{,}000}$
10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}

Again, this table is identical to the previous tables; just move the decimal point left or right to convert the units.

Exercises

29. Convert 2.8 liters to milliliters.

30. Convert 2.8 liters to deciliters.

31. Convert 15 dekaliters to liters.

32. Convert 0.75 deciliters to milliliters.

33. Convert 600 milliliters to centiliters.

34. Convert 600 milliliters to liters.

35. Convert 4.5 deciliters to liters.

36. Convert 550 centiliters to liters.

37. Flying on IcelandAir, you happen to notice that one mini bottle of booze is labeled $50\ \mathrm{mL}$, but another mini bottle is labeled $5\ \mathrm{cL}$. How do the two bottles compare in size?

38. How many 500-milliliter bottles of Coke[1] are equivalent to one 2-liter bottle?

39. The engine displacement of a Yamaha Majesty scooter is 125 cc (cubic centimeters), and the engine displacement of a Chevrolet Spark automobile is 1.4 L (liters). What is the approximate ratio of these engine displacements?

Notes

1. (Fun fact: in Spanish, a 500-milliliter bottle is called a *medio litro.*)

Module 15: Converting Between Systems

You may use a calculator throughout this module.

Converting between the U.S. system and metric system is important in today's global economy; like it or not, the metric system is infiltrating our lives.

The numbers in these conversion ratios are usually difficult to work with, so we will use a calculator whenever necessary and pay attention to rounding. If you discover other conversion ratios that aren't provided here, write them down!

Converting Measurements of Length

You can use the conversion ratios in this table…

1 in = 2.54 cm

1 ft ≈ 0.305 m

1 yd ≈ 0.914 m

1 mi ≈ 1.61 km

...or the equivalent conversion ratios in this table.

1 cm ≈ 0.394 in

1 m ≈ 3.28 ft

1 m ≈ 1.09 yd

1 km ≈ 0.621 mi

Okay, full disclosure: these conversion ratios aren't exactly equivalent. For example, if we reverse the conversion ratio 1 in = 2.54 cm[1], we discover that 1 cm is actually $\frac{1}{2.54} \approx 0.3937$ inches. All of these conversion have been rounded to three significant figures, which means that we may get a slightly different result depending on which version of the conversion ratio we use. As long as we round our answer to three or fewer significant figures, though, we should be all right.

Exercises

1. How many centimeters is 6 inches?

2. Convert 200 yards into meters.

3. 12 feet is equal to how many centimeters?

4. How many feet is 50 meters?

5. 15 meters is what length in yards?

6. Convert 75 centimeters into inches.

7. Is 21 kilometers equivalent to 13 miles? If not, what is the percent error?

8. Is 66 kilometers equivalent to 41 miles? If not, what is the percent error?

Converting Measurements of Weight or Mass

You can use the conversion ratios in this table...

1 oz ≈ 28.35 g

1 lb ≈ 0.454 kg

...or the (almost) equivalent conversion ratios in this table.

1 g ≈ 0.0353 oz

1 kg ≈ 2.20 lb

You probably have noticed that some conversion problems involve multiplication but others involve division. This is based on which version of the conversion ratio you choose; any of these. If you prefer multiplying to dividing, see whether you can figure out a strategy for choosing your conversion ratio.

Exercises

9. Convert 4 ounces into grams.

10. How many kilograms are equivalent to 120 pounds?

11. Convert 50 grams to ounces.

12. Convert 5 kilograms to pounds.

13. How many grams is a half pound of ground beef?

14. In around 2010, the National Collector's Mint (not affiliated with the U.S. Mint) ran a TV commercial selling an imitation \$ 50 gold coin modeled after the U.S. "buffalo" nickel. The commercial made the following claims. *This replica coin is coated in 31 milligrams of pure gold! And the price of gold keeps going up; gold is worth about \$ $1,000$ per ounce! But you can order these fake coins for only \$ 19.95 apiece!* What is the approximate dollar value of the gold in one of these coins?

15. In 2020, the National Collector's Mint is still selling the imitation \$ 50 gold "buffalo" nickel for \$ 19.95. This version of the coin is coated in 14 milligrams of pure gold.[2] As of September 2, 2020, the price of gold is \$ $1,940$ per ounce.[3] What is the approximate dollar value of the gold in one of these coins?

Converting Measurements of Volume or Capacity

You can use the conversion ratios in this table...

1 fl oz ≈ 29.6 mL

1 qt ≈ 0.946 L

1 gal ≈ 3.79 L

...or the (almost) equivalent conversion ratios in this table. This table includes one extra entry.

1 mL ≈ 0.0338 fl oz

1 L ≈ 1.06 qt

1 L ≈ 0.264 gal

1 L ≈ 33.8 fl oz

Exercises

16. A toilet uses 3.5 gallons of water per flush. How many liters is this?

17. How many milliliters of drink are in a 12-ounce can?

18. How many fluid ounces are in a 50 milliliter mini-sized bottle of alcohol?

19. What part of a gallon is in a 2-liter bottle of soda pop?

Converting Measurements: Extensions

Let's finish up with some rates that require conversions.

Exercises

Maxine is driving across Canada. Her car has a 14.2-gallon gas tank and gets an average of 26 miles per gallon.

20. Approximately how many kilometers—actually, the Canadian spelling is kilometres. Approximately how many kilometres can she travel on a full tank of gas?

21. Of course, Canada measures gas in liters. Actually, litres. Convert Maxine's mileage rate, 26 miles per gallon, to kilometres per litre.

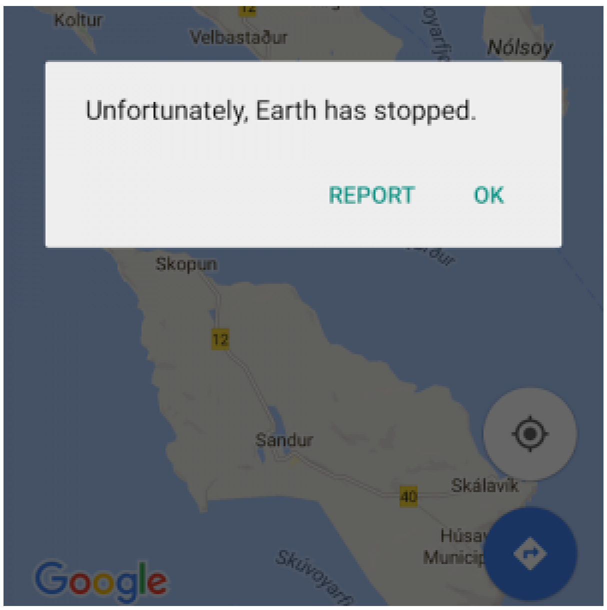
Notes

1. An inch is defined to be exactly 2.54 cm, if you were curious.
2. https://ncmint.com/2020-buffalo-tribute-proof/
3. https://markets.businessinsider.com/commodities/gold-price

Module 16: Other Conversions

You may use a calculator in this module as needed.

Converting Measurements of Time



You probably know all of the necessary conversions for time. When we get to units of time larger than weeks, however, we encounter problems because not all months have the same number of days, a year is not exactly 52 weeks, and the time it takes for the Earth to orbit the Sun is not exactly 365 days. Therefore, it doesn't make sense to expect an exact answer to a question like "how many minutes are in one month?" We will have to use our best judgment in situations such as these.

1 min = 60 sec

1 hr = 60 min

1 day (dy) = 24 hr

1 week (wk) = 7 dy

1 year (yr) = 365 dy

Exercises

1. How many minutes is one standard 365-day year?[1]

2. Have you been alive for 1 billion seconds? Is this even possible?

Converting Rates

Exercises

Usain Bolt holds the world record time for the 100-meter dash, 9.58 seconds.

3. What was his average speed in kilometers per hour?

4. What was his average speed in miles per hour?

The more information we know, the more things we can figure out.

Exercises

An F-15 fighter jet can reach a sustained top speed of roughly Mach 2.3; this is 2.3 times the speed of sound, which is 770 miles per hour.[2]

5. What is the jet's top speed in miles per hour?

6. At this speed, how many miles would the jet travel in one minute?

The jet's range at this speed before it runs out of fuel is around 600 miles.

7. If the jet flies 600 miles at top speed, for how many minutes will it fly?

The jet's maximum fuel capacity is $3,475$ gallons of fuel.

8. If the jet flies 600 miles and burns $3,475$ gallons of fuel, find the jet's fuel efficiency, in miles per gallon.

9. Rewrite the jet's fuel efficiency, in gallons per mile.

10. How many gallons of fuel does the jet consume in one minute?

Measurement Prefixes: Larger

Now let's turn our attention to converting units based on their prefixes. We'll start with large units of measure.

tera- (T)	giga- (G)	mega- (M)	kilo- (k)	**[base unit]**
trillion	billion	million	thousand	one
$1,000,000,000,000$	$1,000,000,000$	$1,000,000$	$1,000$	1
10^{12}	10^{9}	10^{6}	10^{3}	10^{0}

Notice that the powers of these units are multiples of 3, just as with the engineering notation we saw in a previous module. Each prefix is $1,000$ times the next smaller prefix, so moving one place in the chart means moving the decimal point three places. Also notice that capitalization is important; megagram (which is also called a metric ton) is Mg with a capital M, but milligram is mg with a lowercase m.

Using computer memory as an example:

1 kilobyte = $1,000$ bytes

1 megabyte = $1,000$ kilobytes = $1,000,000$ bytes

1 gigabyte = $1,000$ megabytes = $1,000,000$ kilobytes, etc.

1 terabyte = $1,000$ gigabytes = $1,000,000$ megabytes, etc.

Note: There can be inconsistencies with different people's understanding of these prefixes with regards to computer memory, which is counted in powers of 2, not 10. Computer engineers originally defined 1 kilobyte as $1,024$ bytes because $2^{10} = 1,024$, which is very close to $1,000$. However, we will consider these prefixes to be powers of $1,000$, not $1,024$. There is an explanation at https://physics.nist.gov/cuu/Units/binary.html.

Exercises

11. A $5\frac{1}{4}$ inch floppy disk from the 1980s could store about 100 kB; a $3\frac{1}{2}$ inch floppy disk from the 1990s could store about 1.44 MB. By what factor was the storage capacity increased?

12. How many times greater is the storage capacity of a 2 TB hard drive than a 500 GB hard drive?

13. In an article describing small nuclear reactors that are designed to be retrofitted into coal plants, Dr. Jose Reyes of Oregon State University says “One module will produce 60 megawatts of electricity. That’s enough for about 50 thousand homes.”[3] How much electricity per home is this?

14. In the same article, Dr. Reyes says “a 60 megawatt module could produce about 60 million gallons of clean water per day using existing technologies in reverse osmosis.” What is the rate of watts per gallon?

15. The destructive power of nuclear weapons is measured in kilotons (the equivalent of $1,000$ tons of TNT) or megatons (the equivalent of $1,000,000$ tons of TNT). The first nuclear device ever tested, the US’s *Trinity*, was measured at roughly 20 kilotons on July 16, 1945. The largest thermonuclear weapon ever detonated, at 50 megatons, was the USSR’s *Tsar Bomba*, on October 31, 1961.[4] (Video of *Tsar Bomba* was declassified almost 60 years later, in August 2020.) How many times more powerful was *Tsar Bomba* than *Trinity*?

Measurement Prefixes: Smaller

Now we’ll go in the other direction and look at small units of measure.

[base unit]	milli- (m)	micro- (μ or mc)	nano- (n)	pico (p)
one	thousandth	millionth	billionth	trillionth
1	0.001	0.000001	0.000000001	0.000000000001
10^0	10^{-3}	10^{-6}	10^{-9}	10^{-12}

The symbol for micro- is the Greek letter μ (pronounced “myoo”). Because this character can be difficult to replicate, you may see the letter “u” standing in for “μ” in web-based or plaintext technical articles… or you may see the prefix “mc” instead.

Again, the powers are multiples of 3; each prefix gets smaller by a factor of $\frac{1}{1000}$. The negative exponents can sometime be complicated to work with, and it may help to think about things in reverse.

1 meter = 10^3 millimeters = 10^6 micrometers = 10^9 nanometers = 10^{12} picometers

1 second = 10^3 milliseconds = 10^6 microseconds = 10^9 nanoseconds = 10^{12} picoseconds

...and so on.

See https://physics.nist.gov/cuu/Units/prefixes.html for a list of more prefixes.

Exercises

16. An article about network latency compares the following latency times: "So a 10 Mbps link adds 0.4 milliseconds to the RTT, a 100 Mbps link 0.04 ms and a 1 Gbps link just 4 microseconds."[5] Rewrite these times so that they are all in terms of milliseconds, then rewrite them in terms of microseconds.

17. The wavelength of red light is around 700 nm. Infrared radiation has a wavelength of approximately 10 μm.[6] Find the ratio of these wavelengths.

18. Nuclear radiation is measured in units called Sieverts, but because this unit is too large to be practical when discussing people's exposure to radiation, milliSieverts and microSieverts are more commonly used. In 1986, workers cleaning up the Chernobyl disaster were exposed to an estimated dose of 250 mSv.[7] A typical chest x-ray exposes a person to 100 μSv.[8] How many chest x-rays' worth of radiation were the workers exposed to?

Notes

1. If you're familiar with the musical *Rent*, then you already know the answer.
2. My sources for the following set of questions are a combination of former students in the Air National Guard and people who sound like they know what they're talking about on the internet, particularly in this Quora discussion.
3. https://www.kgw.com/article/news/local/oregon-company-get-approval-to-build-nuclear-power-plants/283-7b26b8cd-12d5-4116-928a-065731f7a0f6
4. https://en.wikipedia.org/wiki/Nuclear_weapon_yield
5. https://www.noction.com/blog/network-latency-effect-on-application-performance
6. http://labman.phys.utk.edu/phys222core/modules/m6/The%20EM%20spectrum.html
7. https://en.wikipedia.org/wiki/Chernobyl_disaster
8. https://www.cancer.org/treatment/understanding-your-diagnosis/tests/understanding-radiation-risk-from-imaging-tests.html

Module 17: Angles

You will need a calculator near the end of this module.

Angle measurement is important in construction, surveying, physical therapy, and many other fields. We can visualize an angle as the figure formed when two line segments share a common endpoint. We can also think about an angle as a measure of rotation. A full rotation or a full circle is $360°$, so a half rotation or U-turn is $180°$, and a quarter turn is $90°$.

an angle grinder

We often classify angles by their size.

Acute Angle: between $0°$ and $90°$

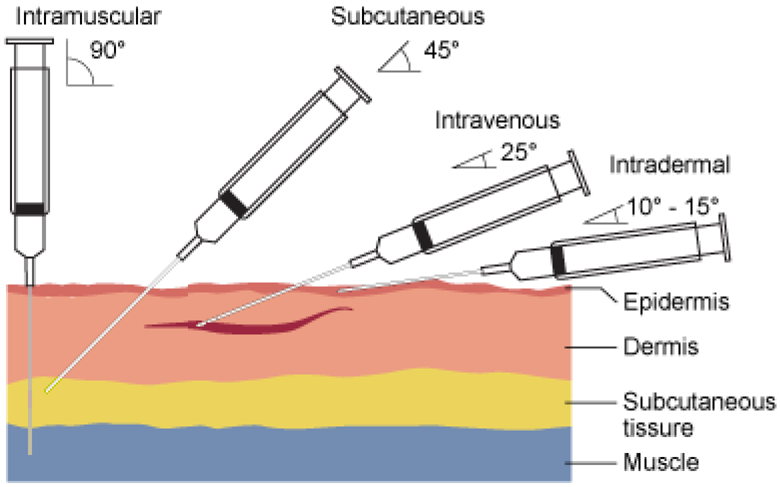
Right Angle: exactly $90°$

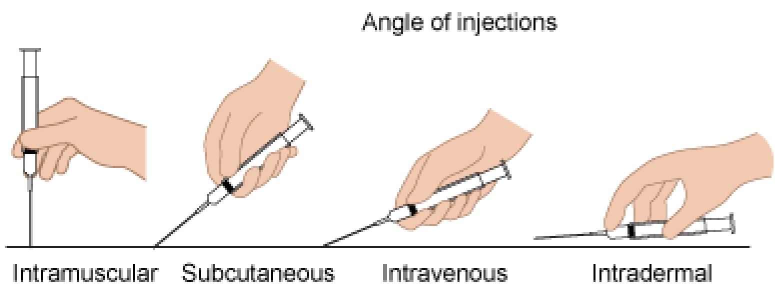
Obtuse Angle: between $90°$ and $180°$

Straight Angle: exactly $180°$

Reflexive Angle: between $180°$ and $360°$

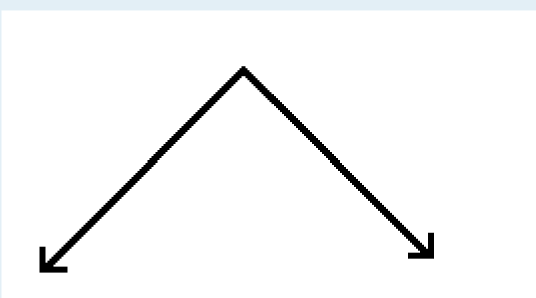
Lines that form a $90°$ angle are called ***perpendicular***. As shown below, the needle should be perpendicular to the body surface for an intramuscular injection.

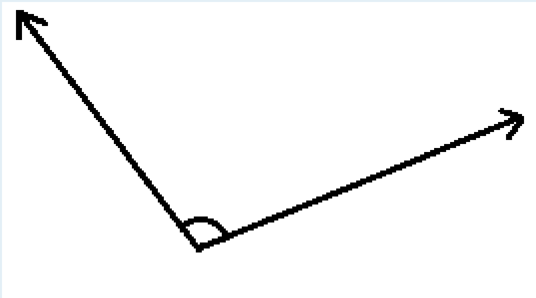


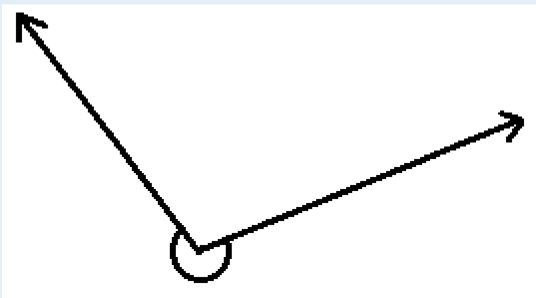


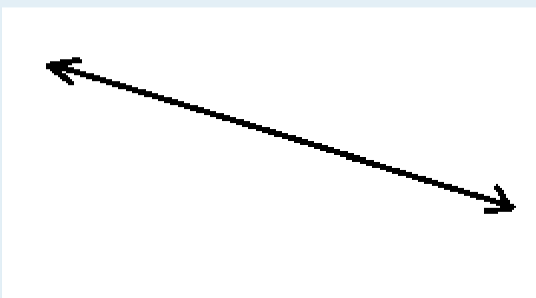
Exercises

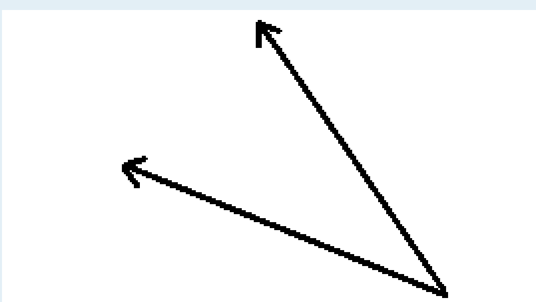
Identify each angle shown below as acute, right, obtuse, straight, or reflexive.

1. 

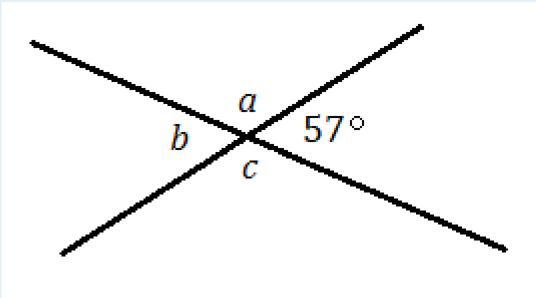
2. 

3. 

4. 

5. 

Find the measure of each unknown angle.

6. 

Angles in Triangles

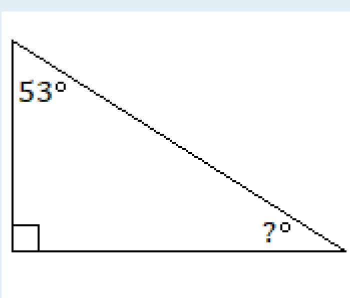
If you need to find the measures of the angles in a triangle, there are a few rules that can help.

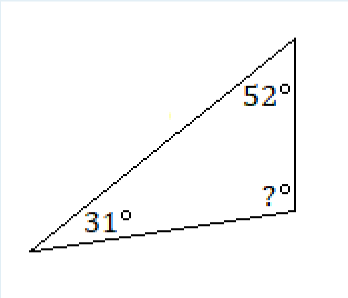
The sum of the angles of every triangle is $180°$.

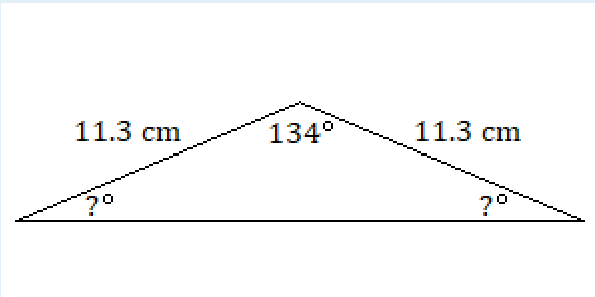
If any sides of a triangle have equal lengths, then the angles opposite those sides will have equal measures.

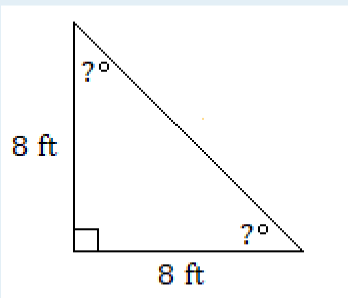
Exercises

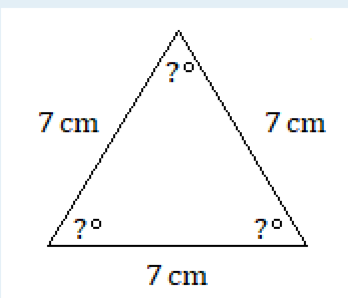
Find the measures of the unknown angles in each triangle.

7. 

8. 

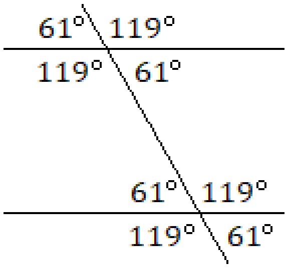
9. 

10. 

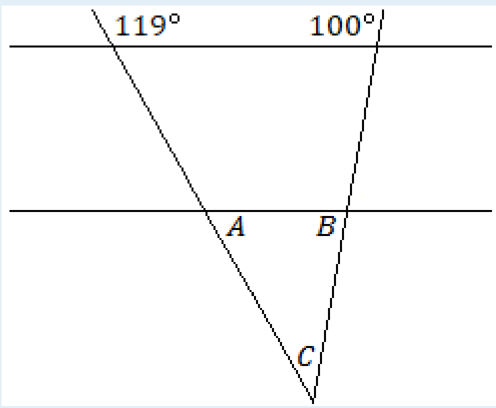
11. 

Angles and Parallel Lines

Two lines that point in the exact same direction and will never cross are called ***parallel*** lines. If two parallel lines are crossed by a third line, sets of equally-sized angles will be formed, as shown in the following diagram. All four acute angles will be equal in measure, all four obtuse angles will be equal in measure, and any acute angle and obtuse angle will have a combined measure of $180°$.



Exercises



12. Find the measures of angles A, B, and C.

Degrees, Minutes, Seconds

It is possible to have angle measures that are not a whole number of degrees. It is common to use decimals in these situations, but the older method—called the ***degrees-minutes-seconds*** or ***DMS*** system—divides a degree using fractions out of 60: a minute is $\frac{1}{60}$ of a degree, and a second is $\frac{1}{60}$ of a minute, which means a second is $\frac{1}{3{,}600}$ of a degree. (Fortunately, these conversions work exactly like time; think of 1 degree as 1 hour.) For example, $2.5° = 2°30'$.

We will look at the procedure for converting between systems, but there are online calculators such as the one at https://www.fcc.gov/media/radio/dms-decimal which will do the conversions for you.

If you have latitude and longitude in DMS, like N $18°54'40''$ W $155°40'51''$, and need to convert it to decimal degrees, the process is fairly simple with a calculator.

Converting from DMS to Decimal Degrees

Enter $\text{degrees} + \text{minutes} \div 60 + \text{seconds} \div 3600$ in your calculator. Round the result to the fourth decimal place, if necessary.[1]

Exercises

Convert each angle measurement from degrees-minutes-seconds into decimal form. Round to the nearest ten-thousandth, if necessary.

13. $67°48'54''$

14. $19°37'25''$

15. $34°14'12''$

Going from decimal degrees to DMS is a more complicated process.

Converting from Decimal Degrees to DMS

1. The whole-number part of the angle measurement gives the number of degrees.
2. Multiply the decimal part by 60. The whole number part of this result is the number of minutes.
3. Multiply the decimal part *of the minutes* by 60. This gives the number of

seconds (including any decimal part of seconds).

For example, let's convert $15.374°$.

1. The *degrees* part of our answer will be 15.
2. The decimal part times 60 is $0.374 \cdot 60 = 22.44$ minutes. The *minutes* part of our answer will be 22.
3. The decimal part times 60 is $0.44 \cdot 60 = 26.4$ seconds. The *seconds* part of our answer will be 26.4.

So $15.374° = 15°22'26.4''$.

Exercises

Convert each angle measurement from decimal into degrees-minutes-seconds form.

16. $26.785°$

17. $58.216°$

18. $41.13°$

Notes

1. We round to four decimal places because 1 second of angle is $\frac{1}{3,600}$ of a degree. This is a smaller fraction than $\frac{1}{1,000}$ so our precision is slightly better than the thousandths place.

Module 18: Triangles

You may use a calculator throughout this module as needed.

Classifying Triangles

We can classify triangles into three categories based on the lengths of their sides.

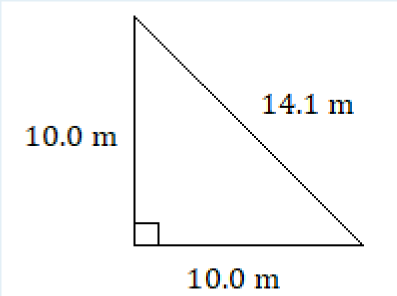
- Equilateral triangle: all three sides have the same length
- Isosceles triangle: exactly two sides have the same length
- Scalene triangle: all three sides have different lengths

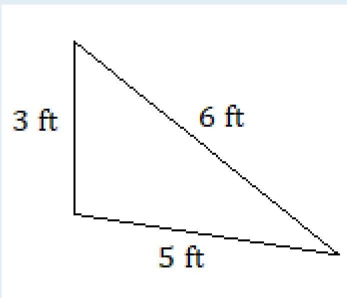
We can also classify triangles into three categories based on the measures of their angles.

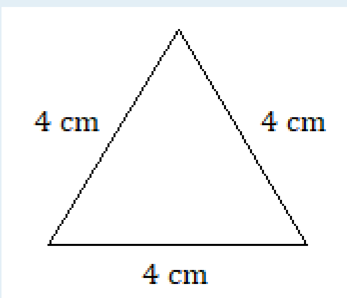
- Obtuse triangle: one of the angles is an obtuse angle
- Right triangle: one of the angles is a right angle
- Acute triangle: all three of the angles are acute

Exercises

Classify each triangle by angle and side. For example, “acute scalene”.)

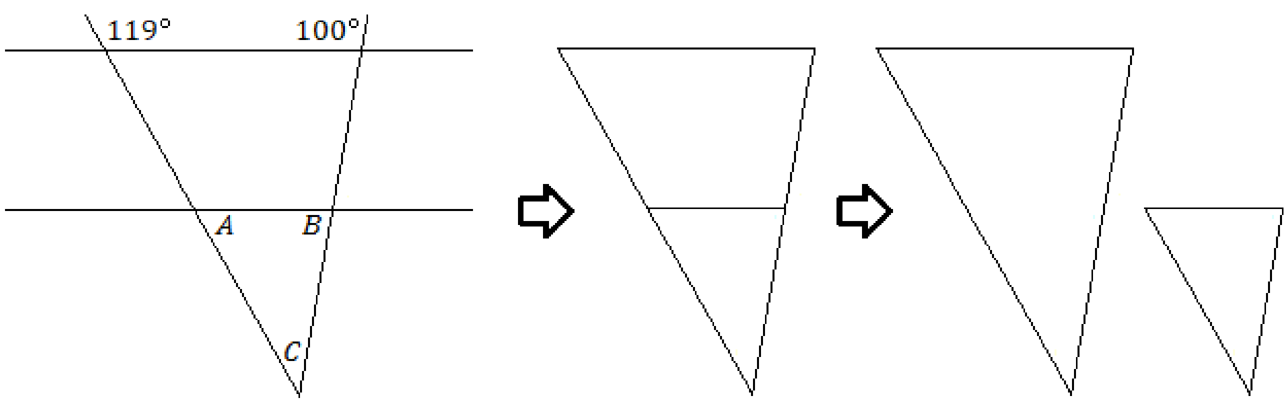
1. 

2. 

3. 

Similar Triangles

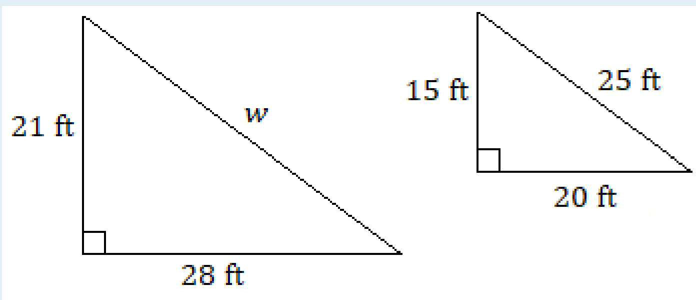
In one of the diagrams in the previous module, the parallel lines included two similar triangles, although they may be hard to see.

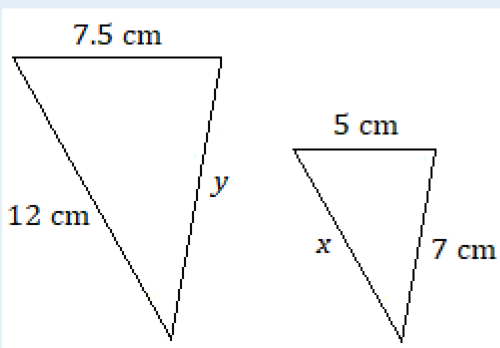


Two triangles are ***similar*** if the three angles of one triangle have the same measure as the three angles of the second triangle. The lengths of the sides of similar triangles will be in the same proportion. The triangles will have the same shape but the lengths will be scaled up or down.

Exercises

Assume that each pair of triangles are similar. Use a proportion to find each unknown length.

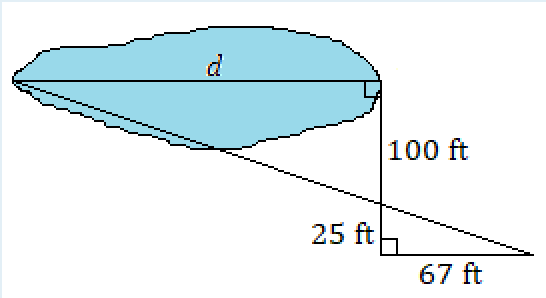
4. 

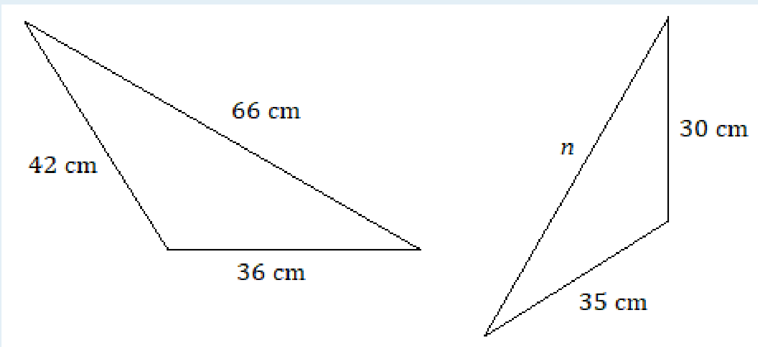
5. 

Recognizing corresponding sides can be more difficult when the figures are oriented differently.

Exercises

Assume that each pair of triangles are similar. Use a proportion to find each unknown length.

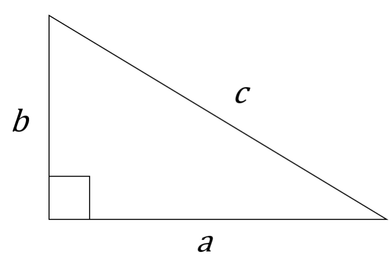
6. 

7. 

Right Triangles

In a right triangle, the two sides that form the right angle are called the ***legs***. The side opposite the right angle, which will always be the longest side, is called the ***hypotenuse***.

The ***Pythagorean theorem*** says that the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.



The Pythagorean Theorem

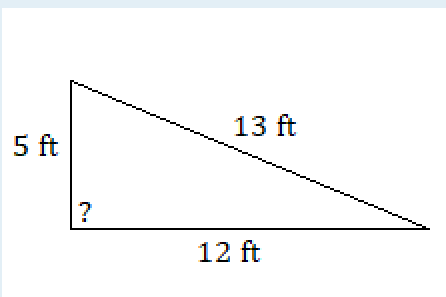
In a right triangle with legs a and b and hypotenuse c,

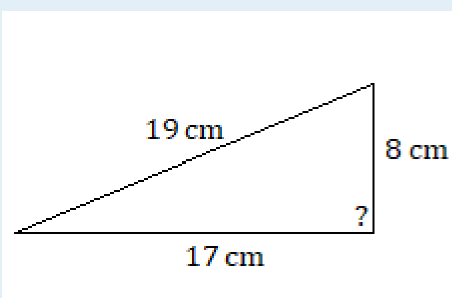
$a^2 + b^2 = c^2$

If you know the lengths of all three sides of a triangle, you can use the Pythagorean theorem to verify whether the triangle is a right triangle or not. The ancient Egyptians used this method for surveying when they needed to redraw boundaries after the yearly flooding of the Nile washed away their previous markings.[1]

Exercises

Use the Pythagorean theorem to determine whether either of the following triangles is a right triangle.

8. 

9. 

Before we continue, we need to briefly discuss square roots. Calculating a square root is the opposite of squaring a number. For example, $\sqrt{49} = 7$ because $7^2 = 49$. If the number under the square root symbol is not a perfect square like 49, then the square root will be an irrational decimal that we will round off as necessary.

Exercises

Use a calculator to find the value of each square root. Round to the hundredths place.

10. $\sqrt{50}$

11. $\sqrt{296}$

12. $\sqrt{943}$

We most often use the Pythagorean theorem to calculate the length of a missing side of a right triangle. Here are three different versions of the Pythagorean theorem arranged to find a missing side, so you don't have to use algebra with $a^2 + b^2 = c^2$.

The Pythagorean Theorem, three other versions

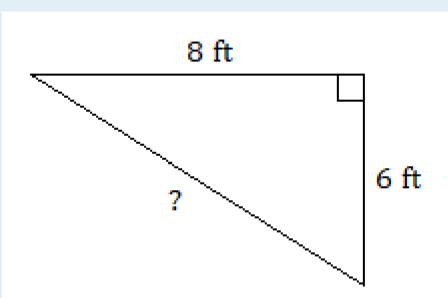
$c = \sqrt{a^2 + b^2}$

$b = \sqrt{c^2 - a^2}$

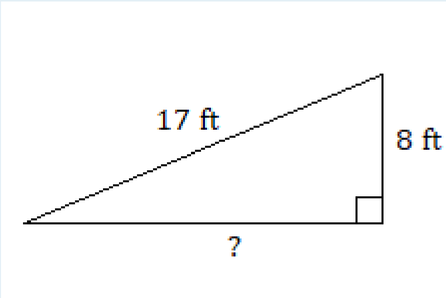
$a = \sqrt{c^2 - b^2}$

Exercises

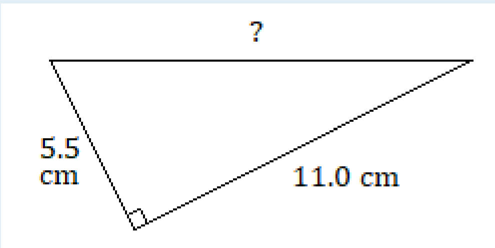
Find the length of the missing side for each of these right triangles. Round to the nearest tenth, if necessary.

13. 

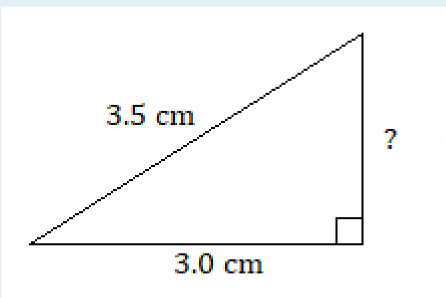
14.



15.



16.

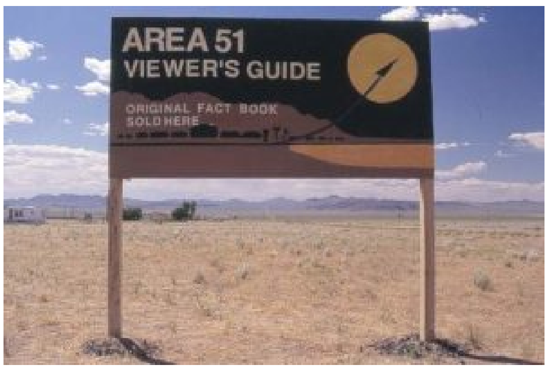


Notes

1. The surveyors were called "rope-stretchers" because they used a loop of rope 12 units long with 12 equally-spaced knots. Three rope-stretchers each held a knot, forming a triangle with lengths 3, 4, and 5 units. When the rope was stretched tight, they knew that the angle between the 3-unit and 4-unit sides was a right angle because $3^2 + 4^2 = 5^2$. From *Discovering Geometry: an Inductive Approach* by Michael Serra, Key Curriculum Press, 1997.

Module 19: Area of Polygons and Circles

You may use a calculator for most of this module as needed.

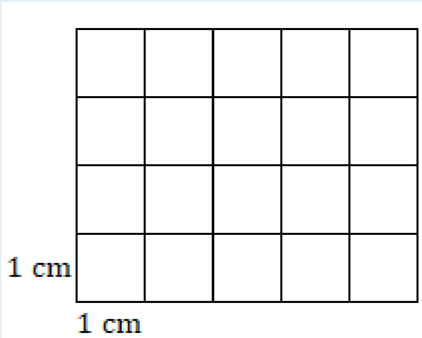


We have seen that the ***perimeter*** of a polygon is the distance around the outside. Perimeter is a length, which is one-dimensional, and so it is measured in linear units (feet, centimeters, miles, etc.). The ***area*** of a polygon is the amount of two-dimensional space inside the polygon, and it is measured in square units: square feet, square centimeters, square miles, etc.

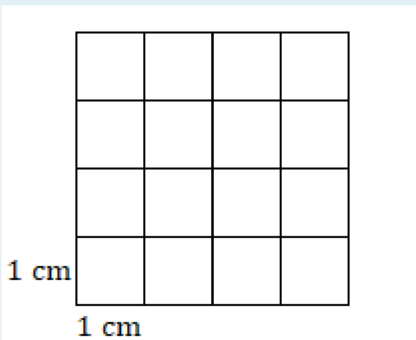
You can always think of area as the number of squares required to completely fill in the shape.

Exercises

1. Find the area of this rectangle.



2. Find the area of this square.



Rectangles and Squares

There are of course formulas for finding the areas of rectangles and squares; we don't have to count little squares.

Area of a Rectangle

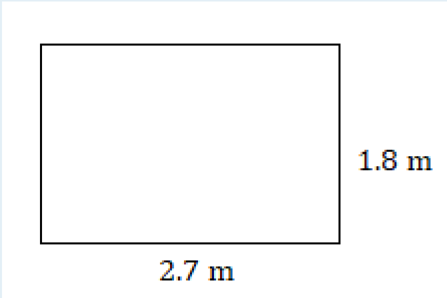
$A = lw$[1] or $A = bh$

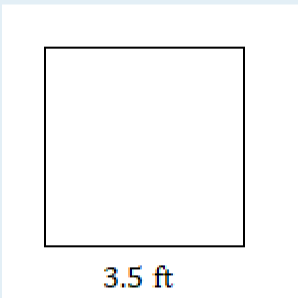
Area of a Square

$A = s^2$

Exercises

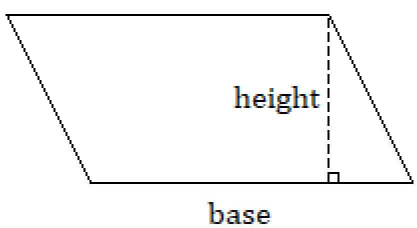
Find the area of each figure.

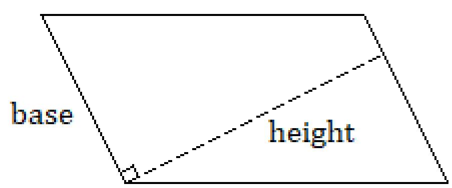
3. 

4. 

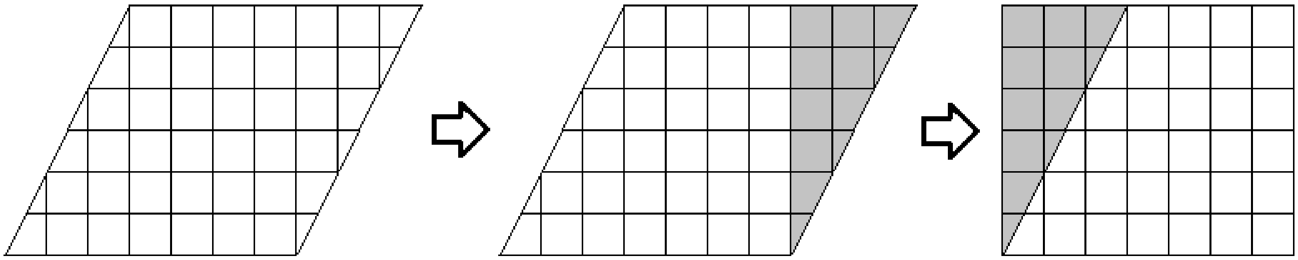
Parallelograms

Another common polygon is the ***parallelogram***, which looks like a tilted rectangle. As the name implies, the pairs of opposite sides are parallel and have the same length. Notice that, if we label one side as a base of the parallelogram, we have a perpendicular height which is *not* the length of the other sides.





The following set of diagrams shows that we can cut off part of a parallelogram and rearrange the pieces into a rectangle with the same base and height as the original parallelogram. A parallelogram with a base of 7 units and a vertical height of 6 units is transformed into a 7 by 6 rectangle, with an area of 42 square units.



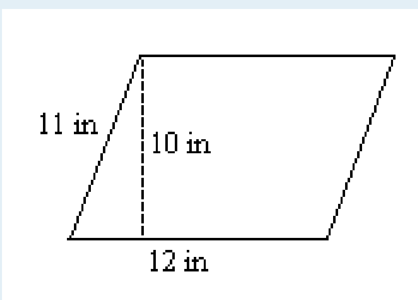
Therefore, the formula for the area of a parallelogram is identical to the formula for the area of a rectangle, provided that we are careful to use the base and the height, which must be perpendicular.

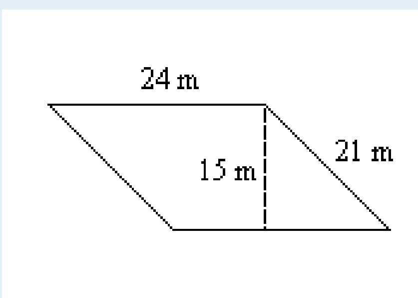
Area of a Parallelogram

$A = bh$

Exercises

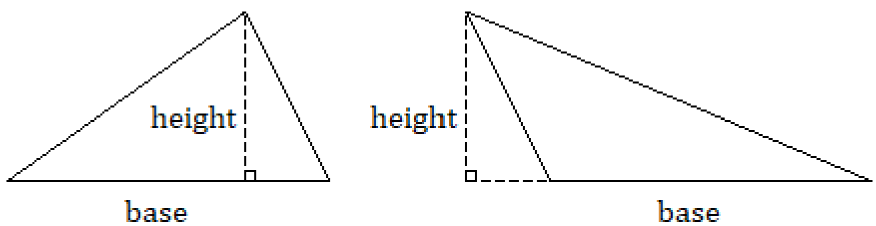
Find the area of each parallelogram.

5. 

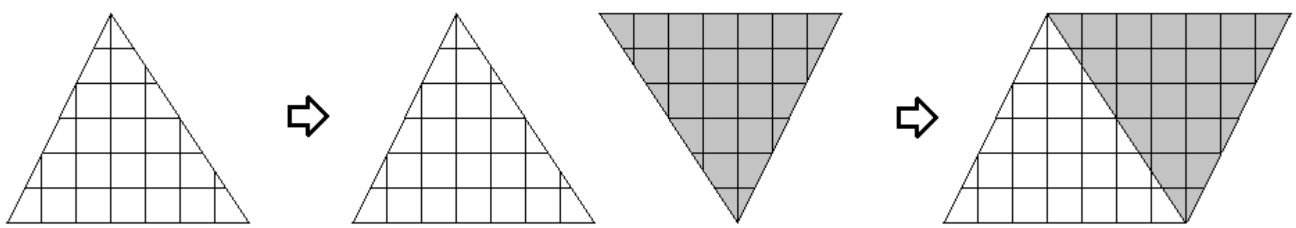
6. 

Triangles

When we need to find the area of a triangle, we need to identify a base and a height that is perpendicular to that base. If the triangle is obtuse, you may have to imagine the height outside of the triangle and extend the base line to meet it.



As shown below, any triangle can be doubled to form a parallelogram. Therefore, the area of a triangle is one half the area of a parallelogram with the same base and height.



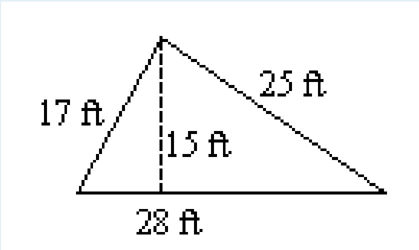
Area of a Triangle

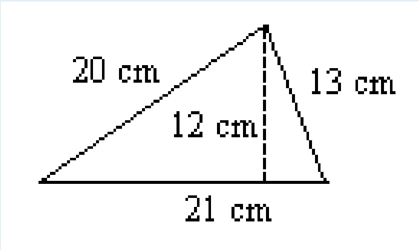
$A = \frac{1}{2}bh$ or $A = bh \div 2$

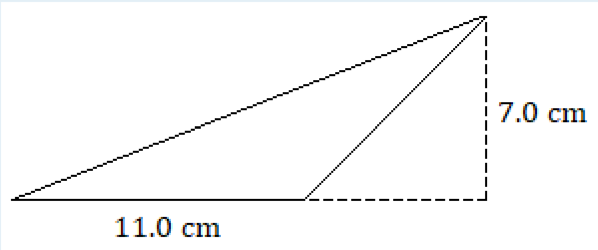
As with a parallelogram, remember that the height must be perpendicular to the base.

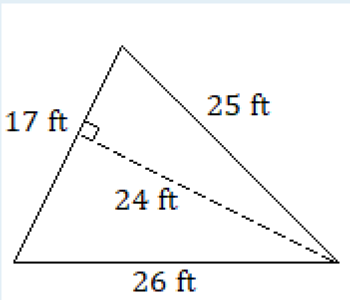
Exercises

Find the area of each triangle.

7. 

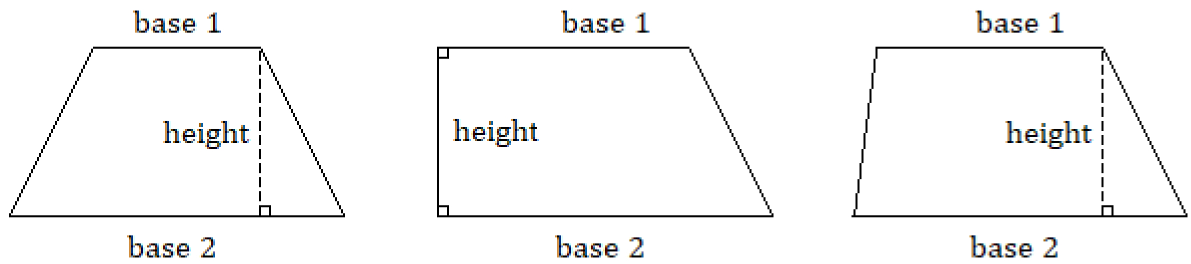
8. 

9. 

10. 

Trapezoids

A somewhat less common quadrilateral is the ***trapezoid***, which has exactly one pair of parallel sides, which we call the bases. The first example shown below is called an isosceles trapezoid because, like an isosceles triangle, its two nonparallel sides have equal lengths.



There are a number of ways to show where the area formula comes from, but the explanations are better in video because they can be animated.[234]

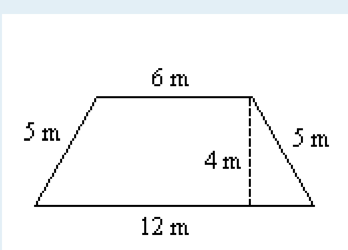
Area of a Trapezoid

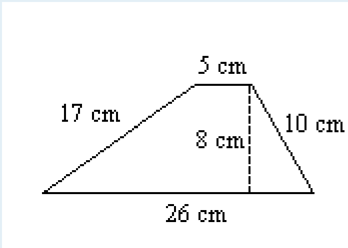
$A = \frac{1}{2}h(b_1 + b_2)$ or $A = (b_1 + b_2)h \div 2$

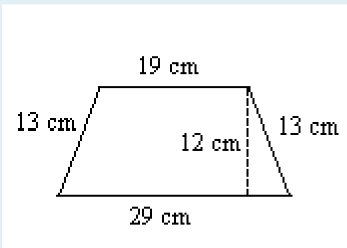
Don't be intimidated by the subscripts on b_1 and b_2; it's just a way to name two different measurements using the same letter for the variable. (Many people call the bases a and b instead; feel free to write it whichever way you prefer.) Whatever you call them, you just add the two bases, multiply by the height, and take half of that.

Exercises

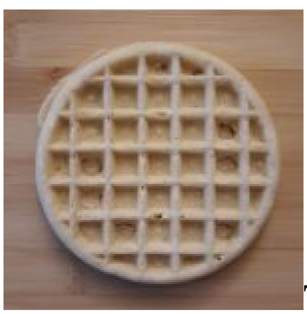
Find the area of each trapezoid.

11. 

12. 

13. 

Circles



The area of a circle is π times the square of the radius: $A = \pi r^2$. The units are still square units, even though a circle is round. (Think of the squares on a round waffle.) Because we can't fit a whole number of squares—or an exact fraction of squares—inside the circle, the area of a circle will be an approximation.

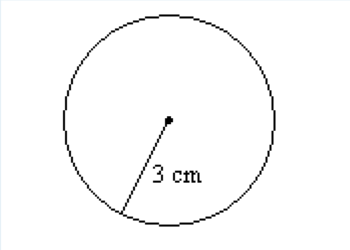
Area of a Circle

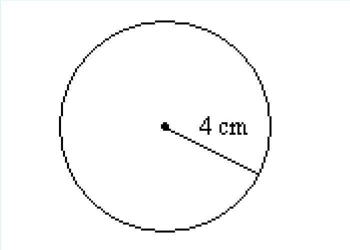
$A = \pi r^2$

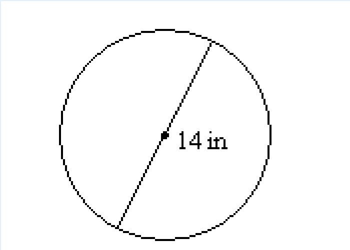
Remember that $\pi \approx 3.1416$.

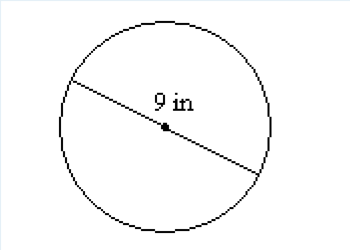
Exercises

Find the area of each circle. Round to the nearest tenth or to three significant figures, whichever seems appropriate.

14. 

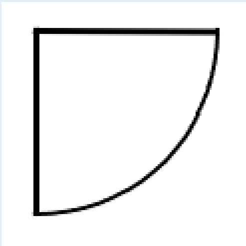
15. 

16. 

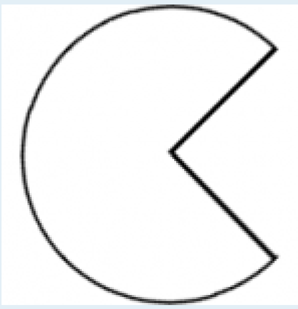
17. 

Each figure is a fraction of a circle. Calculate each area.

18. The radius of the quarter circle is 5 meters.



19. A quarter circle has been removed from a circle with a diameter of 7 feet.



Notes

1. You might choose to use capital letters for the variables here because a lowercase letter "l" can easily be mistaken for a number "1".
2. https://youtu.be/yTnYRpcZA9c
3. https://youtu.be/WZtO3oERges
4. https://youtu.be/uLHc6Br2veg

Module 20: Composite Figures

You may use a calculator throughout this module as needed.

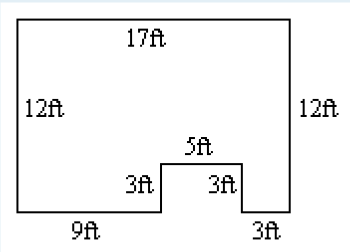
Many objects have odd shapes made up of simpler shapes. A ***composite figure*** is a geometric figure which is formed by—or *composed* of—two or more basic geometric figures. We will look at a handful of fairly simple examples, but this concept can of course be extended to much more complicated figures.

This sunburst pattern is composed of 276 geometric tiles.

To find the area of a composite figure, it is generally a good idea to divide it into simpler shapes and either add or subtract their areas as necessary.

Exercises

A floor plan of a room is shown. The room is a 12-foot by 17-foot rectangle, with a 3-foot by 5-foot rectangle cut out of the south side.

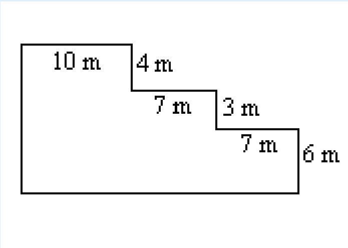


1. Determine the amount of molding required to go around the perimeter of the room.
2. Determine the amount of flooring required to cover the entire area.

You may need to figure out some unmarked dimensions.

Exercises

A plan for an irregular parking lot is shown.



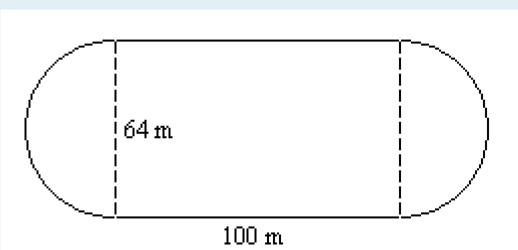
3. Calculate the perimeter.

4. Calculate the area.

Sometimes, composite figures involve parts of circles coupled with polygons.

Exercises

A high school is building a track at its athletic fields. The track, which is formed by two straight sides and two semicircles as shown in the plans below, is supposed to have a total length of 400 meters.

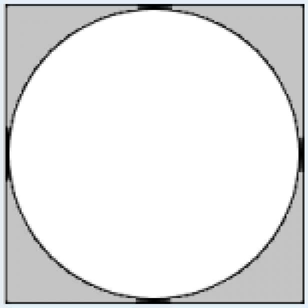


5. Determine the distance around the track. Will the track be the right length?

6. After the new track is built, landscapers need to lay sod on the field inside the track. What is the area of the field inside the track?

If we need to determine a fraction or percent out of the whole, we may be able to solve the problem without knowing any actual measurements.

Exercises



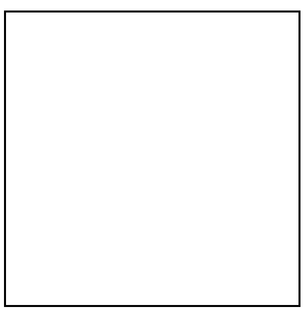
7. Circular disks are being cut from squares of sheet metal, with the remainder around the corners being discarded. Assuming that the circles are made as large as possible, what percent of the sheet metal will be discarded?

Module 21: Converting Units of Area

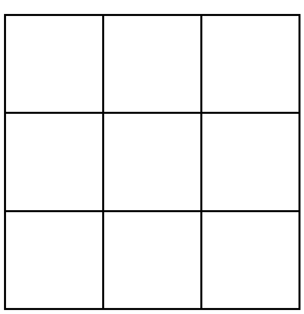
You may use a calculator throughout this module.

Converting between units of area requires us to be careful because square units behave differently than linear units.

U.S. System: Converting Measurements of Area



Consider a square yard; the area of a square with sides 1 yard long.



1 yard = 3 feet, so we can divide the square into three sections vertically and three sections horizontally to convert both dimensions of the square from yards to feet. This forms a 3 by 3 grid, which shows us visually that 1 square yard equals 9 square feet, not 3 square feet! The linear conversion ratio of 1 to 3 means that that the conversion ratio for the areas is 1 to 3^2, or 1 to 9.

Here's another way to think about it without a diagram: $1\ \mathrm{yd} = 3\ \mathrm{ft}$, so $(1\ \mathrm{yd})^2 = (3\ \mathrm{ft})^2$. To remove the parentheses, we must square the number *and* square the units: $(3\ \mathrm{ft})^2 = 3^2\ \mathrm{ft}^2 = 9\ \mathrm{ft}^2$.

More generally, we need to **square** the linear conversion factors when converting units of area. If the linear units have a ratio of 1 to n, the square units will have a ratio of 1 to n^2.

Exercises

1. An acre is defined as the area of a 660 foot by 66 foot rectangle. (That's a furlong by a chain, if you were curious.) How many square feet are in 1 acre?
2. How many square yards are in 1 acre?
3. How many square inches equals 1 square foot?

It should be no surprise that this module will be full of conversion ratios. As always, if you discover other conversion ratios that aren't provided here, it would be a good idea to write them down so you can use them as needed.

$1\ \mathrm{ft}^2 = 144\ \mathrm{in}^2$

$1\ \mathrm{yd}^2 = 9\ \mathrm{ft}^2$

$1\ \mathrm{acre}\ (\mathrm{ac}) = 43,560\ \mathrm{ft}^2$

$1\ \mathrm{ac} = 4,840\ \mathrm{yd}^2$

$1\ \mathrm{mi}^2 = 27,878,400\ \mathrm{ft}^2$

$1\ \mathrm{mi}^2 = 3,097,600\ \mathrm{yd}^2$

$1\ \mathrm{mi}^2 = 640\ \mathrm{ac}$

An acre is defined as a unit of area; it would be wrong to say "acres squared" or put an exponent of 2 on the units.

Exercises

4. A hallway is 9 yards long and 2 yards wide. How many square feet of linoleum are needed to cover the hallway?

5. A proposed site for an elementary school is 600 feet by 600 feet. Find its area, in acres.

Metric System: Converting Measurements of Area

$$1 \text{ cm}^2 = 100 \text{ mm}^2$$

$$1 \text{ m}^2 = 1,000,000 \text{ mm}^2$$

$$1 \text{ m}^2 = 10,000 \text{ cm}^2$$

$$1 \text{ hectare (ha)} = 10,000 \text{ m}^2$$

$$1 \text{ km}^2 = 1,000,000 \text{ m}^2$$

$$1 \text{ km}^2 = 100 \text{ ha}$$

A hectare is defined as a square with sides 100 meters long. Dividing a square kilometer into ten rows and ten columns will make a 10 by 10 grid of 100 hectares. As with acres, it would be wrong to say "hectares squared" or put an exponent of 2 on the units.

Exercises

6. A hallway is 9 meters long and 2 meters wide. How many square centimeters of linoleum are needed to cover the hallway?

7. A proposed site for an elementary school is 200 meters by 200 meters. Find its area, in hectares.

Both Systems: Converting Measurements of Area

Converting between the U.S. and metric systems will involve messy decimal values. For example, because $1 \text{ in} = 2.54 \text{ cm}$, we can square both numbers and find that $(1 \text{ in})^2 = (2.54 \text{ cm})^2 = 6.4516 \text{ cm}^2$. The conversions are rounded to three or four significant digits in the table below.

$$1 \text{ in}^2 \approx 6.45 \text{ cm}^2 \leftrightarrow 1 \text{ cm}^2 \approx 0.155 \text{ in}^2$$

$$1 \text{ in}^2 \approx 6.45 \text{ cm}^2 \leftrightarrow 1 \text{ cm}^2 \approx 0.155 \text{ in}^2$$

$$1 \text{ yd}^2 \approx 0.836 \text{ m}^2 \leftrightarrow 1 \text{ m}^2 \approx 1.196 \text{ yd}^2$$

$$1 \text{ mi}^2 \approx 2.59 \text{ km}^2 \leftrightarrow 1 \text{ km}^2 \approx 0.386 \text{ mi}^2$$

$$1 \text{ ac} \approx 0.405 \text{ ha} \leftrightarrow 1 \text{ ha} \approx 2.47 \text{ ac}$$

Exercises

8. The area of Portland is 145 mi^2. Convert this area to square kilometers.

9. How many hectares is a $5,000$ acre ranch?

10. A sheet of paper measures 8.5 inches by 11 inches. What is the area in square centimeters?

11. A soccer field is 100 meters long and 70 meters wide. What is its area in square feet?

Areas of Similar Figures

Earlier in this module, it was stated that if the linear units have a ratio of 1 to n, the square units will have a ratio of 1 to n^2. This applies to similar figures as well.

> If the linear dimensions of two similar figures have a ratio of 1 to n, then the areas will have a ratio of 1 to n^2.

This is true for circles, similar triangles, similar rectangles, similar hexagons, you name it. We'll verify this in the following exercises.

Exercises

A personal pizza has a 7-inch diameter. A medium pizza has a diameter twice that of a personal pizza.

12. Determine the area of the medium pizza.

13. Determine the area of the personal pizza.

14. What is the ratio of the areas of the two pizzas?

Right triangle ABC has legs 3 cm and 4 cm long. Right triangle DEF has legs triple the length of ABC's.

15. Determine the area of the larger triangle, DEF.

16. Determine the area of the smaller triangle, ABC.

17. What is the ratio of the areas of the two triangles?

Module 22: Surface Area of Common Solids

You may use a calculator throughout this module.

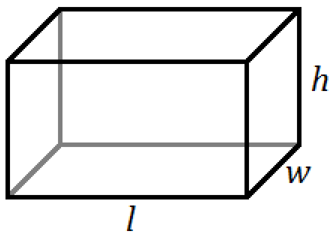
We will now turn our attention from two-dimensional figures to three-dimensional figures, which we often call ***solids***, even if they are hollow inside.

In this module, we will look the surface areas of some common solids. (We will look at volume in a later module.) Surface area is what it sounds like: it's the sum of the areas of all of the outer surfaces of the solid. When you are struggling to wrap a present because your sheet of wrapping paper isn't quite big enough, you are dealing with surface area.

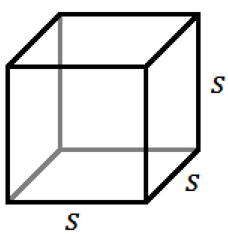
There are two different kinds of surface area that are important: the ***lateral surface area*** (LSA) and ***total surface area*** (TSA).

To visualize the difference between LSA and TSA, consider a can of soup. The lateral surface area would be used to measure the size of the paper label around the can. The total surface area would be used to measure the amount of sheet metal needed to make the can. In other words, the total surface area includes the top and bottom, whereas the lateral surface area does not.

Rectangular Solids



A ***rectangular solid*** looks like a rectangular box. It has three pairs of equally sized rectangles on the front and back, on the left and right, and on the top and bottom.



A ***cube*** is a special rectangular solid with equally-sized squares for all six faces.

The lateral surface area is the combined total area of the four vertical faces of the solid, but not the top and bottom. If you were painting the four walls of a room, you would be thinking about the lateral surface area.

The total surface area is the combined total area of all six faces of the solid. If you were painting the four walls, the floor, and the ceiling of a room, you would be thinking about the total surface area.

For a rectangular solid with length l, width w, and height h...[1]

$$LSA = 2lh + 2wh$$

$$TSA = 2lh + 2wh + 2lw$$

$$TSA = LSA + 2lw$$

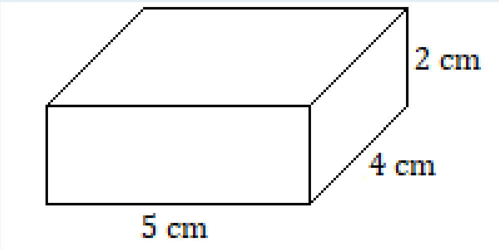
For a cube with side length s...

$$LSA = 4s^2$$

$$TSA = 6s^2$$

Note: These dimensions are sometimes called base, depth, and height.

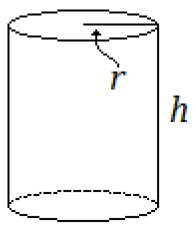
Exercises



1. Find the lateral surface area of this rectangular solid.

2. Find the total surface area of this rectangular solid.

Cylinders



As mentioned earlier in this module, the lateral surface area of a soup can is the paper label, which is a rectangle. Therefore, the lateral surface area of a ***cylinder*** is a rectangle; its width is equal to the circumference of the circle, $2\pi r$, and its height is the height of the cylinder.

Since a cylinder has equal-sized circles at the top and bottom, its total surface area is equal to the lateral surface area plus twice the area of one of the circles.

For a cylinder with radius r and height h...

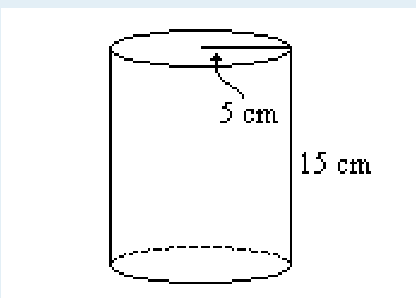
$LSA = 2\pi rh$

$TSA = 2\pi rh + 2\pi r^2$

$TSA = LSA + 2\pi r^2$

Be aware that if you are given the diameter of the cylinder, you will need to cut it in half before using these formulas.

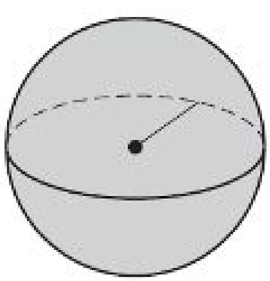
Exercises



3. Find the lateral surface area of this cylinder.

4. Find the total surface area of this cylinder.

Spheres



The final solid of this module is the ***sphere***, which can be thought of as a circle in three dimensions: every point on the surface of a sphere is the same distance from the center. Because of this, a sphere has only one important measurement: its radius. Of course, its diameter could be important also, but the idea is that a sphere doesn't have different dimensions such as length, width, and height. A sphere has the same radius (or diameter) in every direction.

We would need to use calculus to derive the formula for the surface area of a sphere, so we'll just assume it's true and get on with the business at hand. Notice that, because a sphere doesn't have top or bottom faces, we don't need to worry about finding the lateral surface area. The only surface area is the total surface area.

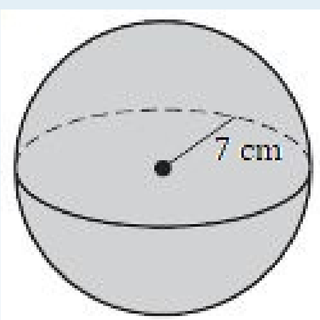
For a sphere with radius r or diameter d...

$SA = 4\pi r^2$ or $SA = \pi d^2$

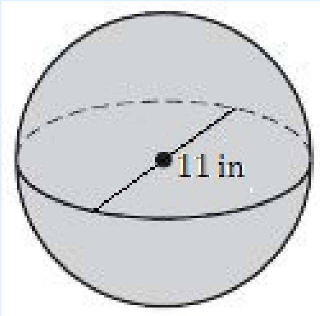
Coincidentally, the surface area of a sphere is 4 times the area of the cross-sectional circle at the sphere's widest part. You may find it interesting to try to visualize this, or head to the kitchen for a demonstration: if you cut an orange into four quarters, the peel on one of those quarter oranges has the same area as the circle formed by the first cut.

Exercises

5. Find the surface area of this sphere.



6. Find the surface area of this sphere.



Notes

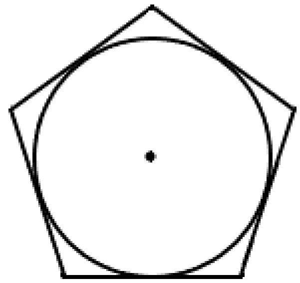
1. You might choose to use capital letters for the variables here because a lowercase letter "l" can easily be mistaken for a number "1".

Module 23: Area of Regular Polygons

You may use a calculator throughout this module.

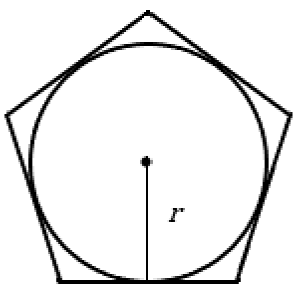
The Pentagon building spans 28.7 acres ($116,000\ \mathrm{m}^2$), and includes an additional 5.1 acres ($21,000\ \mathrm{m}^2$) as a central courtyard.[1] A pentagon is an example of a regular polygon.

A ***regular polygon*** has all sides of equal length and all angles of equal measure. Because of this symmetry, a circle can be ***inscribed***—drawn inside the polygon touching each side at one point—or ***circumscribed***—drawn outside the polygon intersecting each vertex. We'll focus on the inscribed circle first.



inscribed circle

Let's call the radius of the inscribed circle lowercase r; this is the distance from the center of the polygon perpendicular to one of the sides.[2]



Area of a Regular Polygon (with a radius drawn to the center of one side)[3]

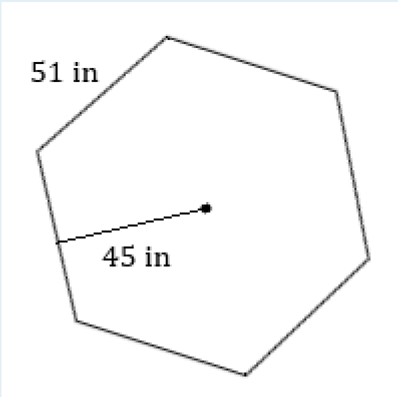
For a regular polygon with n sides of length s, and inscribed (inner) radius r,

$A = nsr \div 2$

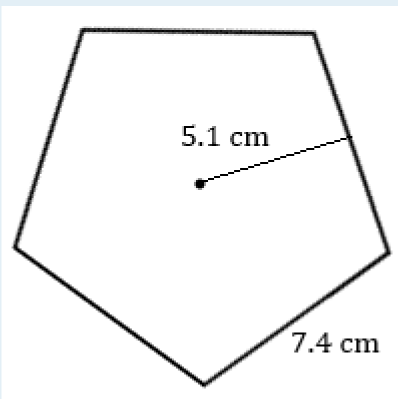
Note: This formula is derived from dividing the polygon into n equally-sized triangles and combining the areas of those triangles.

Exercises

1. Calculate the area of this regular hexagon.

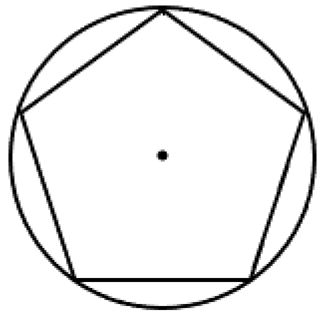


2. Calculate the area of this regular pentagon.



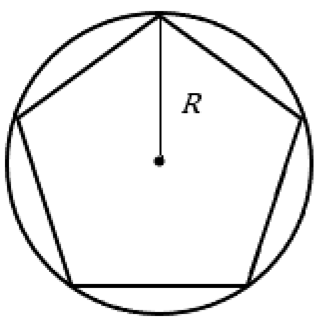
3. A stop sign has a a height of 30 inches, and each edge measures 12.5 inches. Find the area of the sign.

Okay, but what if we know the distance from the center to one of the corners instead of the distance from the center to an edge? We'll need to imagine a circumscribed circle.



circumscribed circle

Let's call the radius of the circumscribed circle capital R; this is the distance from the center of the polygon to one of the vertices (corners).



Area of a Regular Polygon (with a radius drawn to a vertex)[4]

For a regular polygon with n sides of length s, and circumscribed (outer) radius R,

$$A = 0.25ns\sqrt{4R^2 - s^2}$$

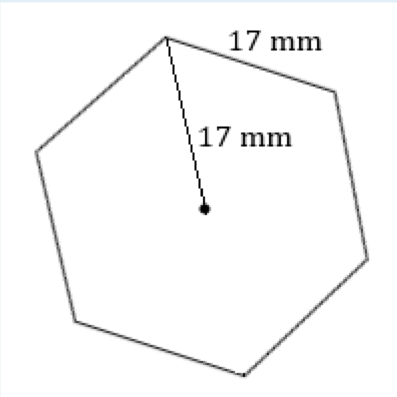
or

$$A = ns\sqrt{4R^2 - s^2} \div 4$$

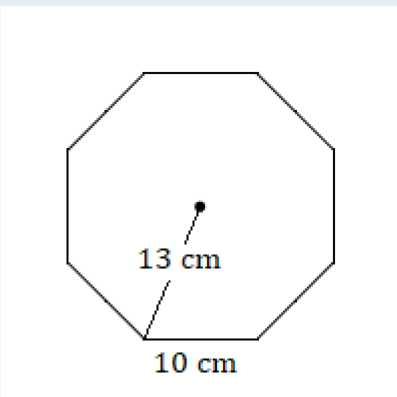
Note: This formula is also derived from dividing the polygon into n equally-sized triangles and combining the areas of those triangles. This formula includes a square root because it involves the Pythagorean theorem.

Exercises

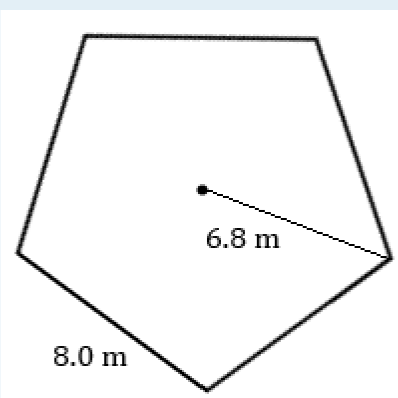
4. Calculate the area of this regular hexagon.



5. Calculate the area of this regular octagon.

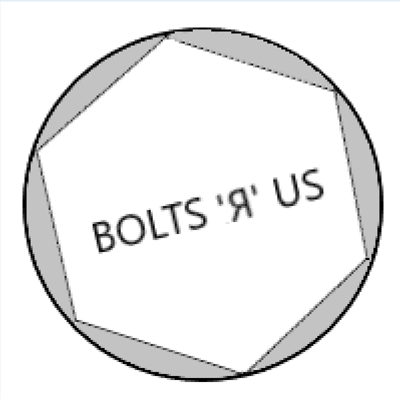


6. Calculate the area of this regular pentagon.



As you know, a ***composite figure*** is a geometric figure which is formed by joining two or more basic geometric figures. Let's look at a composite figure formed by a circle and a regular polygon.

Exercises



7. The hexagonal head of a bolt fits snugly into a circular cap with a circular hole with inside diameter 46 mm as shown in this diagram. Opposite sides of the bolt head are 40 mm apart. Find the total empty area in the hole around the edges of the bolt head.

Notes

1. https://en.wikipedia.org/wiki/The_Pentagon
2. The inner radius is more commonly called the *apothem* and labeled a, but we are trying to keep the jargon to a minimum in this textbook.
3. This formula is more commonly written as one-half the apothem times the perimeter: $A = \frac{1}{2}ap$
4. Your author created this formula because every other version of it uses trigonometry, which we aren't covering in this textbook.

Module 24: Volume of Common Solids

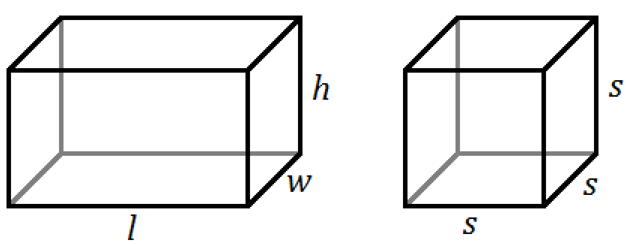
You may use a calculator throughout this module.

Note: We will not necessarily follow the rules for rounding (precision and accuracy) in this module. Many of these figures have dimensions with only one significant figure, but we would lose a lot of information if we rounded the results to only one sig fig.

In the answer key, we will often round to the nearest whole number, or to the nearest tenth, or to two or three significant figures as we deem appropriate.

Wanda Ortiz of The Iron Maidens turns up the volume.

The surface area of a solid is the sum of the areas of all its faces; therefore, surface area is two-dimensional and measured in square units. The ***volume*** is the amount of space inside the solid. Volume is three-dimensional, measured in cubic units. You can imagine the volume as the number of cubes required to completely fill up the solid.



Volume of a Rectangular Solid

For a rectangular solid with length l, width w, and height h:

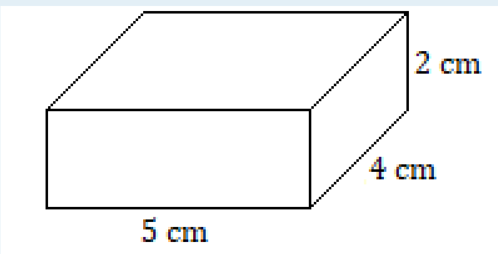
$$V = lwh$$

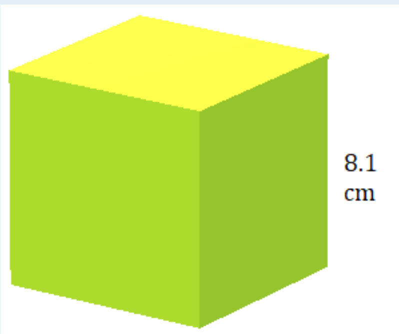
For a cube with side length s:

$$V = s^3$$

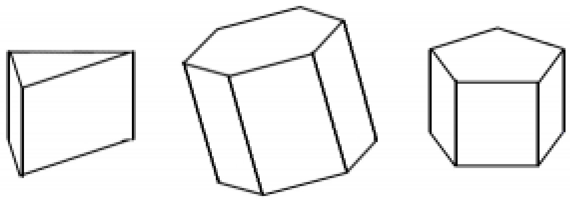
Exercises

Find the volume of each solid.

1. 

2. 

A solid with two equal sized polygons as its bases and rectangular lateral faces is called a right-angle prism. Some examples are shown below. We will refer to them simply as ***prisms*** in this textbook. (We will not be working with oblique prisms, which have parallelograms for the lateral faces.)



If you know the area of one of the bases, multiplying it by the height gives you the volume of the prism. In the formula below, we are using a capital B to represent the *area* of the base.

Volume of a Prism

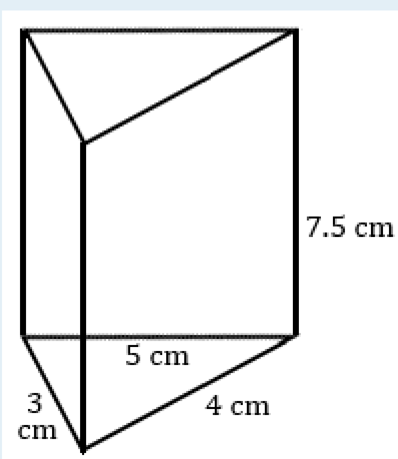
For a prism with base area B and height h:

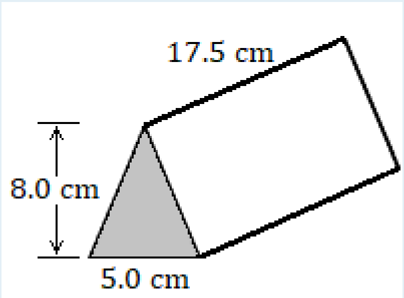
$$V = Bh$$

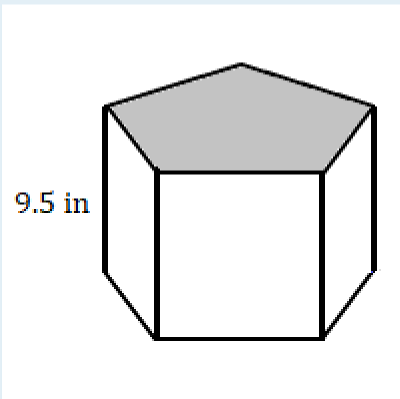
If the prism is lying on its side, the "height" will look like a length. No matter how the prism is oriented, the height is the dimension that is perpendicular to the planes of the two parallel bases.

Exercises

Find the volume of each prism.

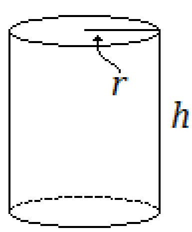
3. 

4. 

5. 

The area of the pentagon is 55 square inches.

A cylinder can be thought of as a prism with bases that are circles, rather than polygons. Just as with a prism, the volume is the area of the base multiplied by the height.



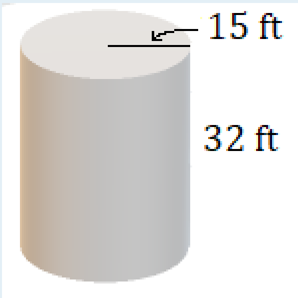
Volume of a Cylinder

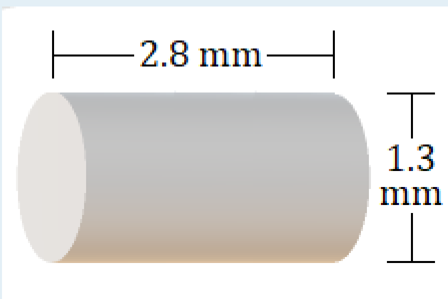
For a cylinder with radius r and height h:

$V = \pi r^2 h$

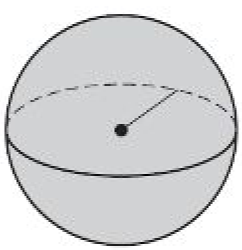
Exercises

Find the volume of each cylinder.

6. 

7. 

As with surface area, we would need to use calculus to derive the formula for the volume of a **sphere**. Just believe it. ¯_(ツ)_/¯



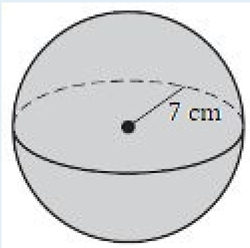
Volume of a Sphere

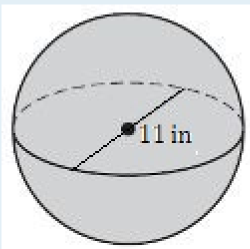
For a sphere with radius r:

$V = \frac{4}{3}\pi r^3$ or $V = 4\pi r^3 \div 3$

Exercises

Find the volume of each sphere.

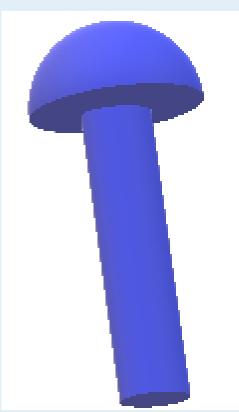
8. 

9. 

Composite Solids

Of course, not every three-dimensional object is a prism, cylinder, or sphere. A composite solid is made up of two or more simpler solids. As with two-dimensional composite figures, breaking the figure into recognizable solids is a good first step.

Exercises



10. A rivet is formed by topping a cylinder with a hemisphere. The width of the cylindrical part (the rivet pin) is 1.6 cm and the length is 7 cm. The width of the hemisphere-shaped top (the rivet head) is 3.2 cm. Find the rivet's volume.

Module 25: Converting Units of Volume

You may use a calculator throughout this module.

Just as we saw with area, converting between units of volume requires us to be careful because cubic units behave differently than linear units.

Quantities of mulch, dirt, or gravel are often measured by the cubic yard. How many cubic feet are in one cubic yard?

1 yard = 3 feet, so we can divide the length into three sections, the width into three sections, and the height into three sections to convert all three dimensions of the cube from yards to feet. This forms a 3 by 3 by 3 cube, which shows us that 1 cubic yard equals 27 cubic feet. The linear conversion ratio of 1 to 3 means that that the conversion ratio for the volumes is 1 to 3^3, or 1 to 27.

Here's another way to think about it without a diagram: $1 \text{ yd} = 3 \text{ ft}$, so $(1 \text{ yd})^3 = (3 \text{ ft})^3$. To remove the parentheses, we must cube the number *and* cube the units: $(3 \text{ ft})^3 = 3^3 \text{ ft}^3 = 27 \text{ ft}^3$.

More generally, we need to **cube** the linear conversion factors when converting units of volume. If the linear units have a ratio of 1 to n, the cubic units will have a ratio of 1 to n^3.

Exercises

1. Determine the number of cubic inches in 1 cubic foot.

2. Determine the number of cubic inches in 1 cubic yard.

3. Determine the number of cubic millimeters in 1 cubic centimeter.

4. Determine the number of cubic centimeters in 1 cubic meter.

U.S. System: Converting Measurements of Volume

$$1\text{ ft}^3 = 1,728\text{ in}^3$$
$$1\text{ yd}^3 = 27\text{ ft}^3$$
$$1\text{ yd}^3 = 46,656\text{ in}^3$$

Exercises

5. True story: A friend at the National Guard base gave us three long wooden crates to use as raised planting beds. (The crates probably carried some kind of weapons or ammunition, but our friend wouldn't say.) Henry, who was taking geometry in high school, was asked to measure the crates and figure out how much soil we needed. The inside dimensions of each crate were 112 inches long, 14 inches wide, and 14 inches deep. We wanted to fill them most of the way full with soil, leaving about 4 inches empty at the top. How many cubic yards of soil did we need to order from the supplier?

6. True story, continued: I decided to check our answer and did a rough estimate by rounding each dimension to the nearest foot, then figuring out the volume from there. Did this give the same result?

We can convert between units of volume and liquid capacity. As you might expect, the numbers are messy in the U.S. system.

$$1 \text{ fl oz} \approx 1.805 \text{ in}^3 \leftrightarrow 1 \text{ in}^3 \approx 0.554 \text{ fl oz}$$
$$1 \text{ ft}^3 \approx 7.48 \text{ gal} \leftrightarrow 1 \text{ gal} \approx 0.1337 \text{ ft}^3$$

Exercises

7. A wading pool has a diameter of roughly 5 feet and a depth of 6 inches. How many gallons of water are required to fill it about 80% of the way full?

8. A standard U.S. soda pop can has a diameter of $2\frac{1}{2}$ inches and a height of $4\frac{3}{4}$ inches. Verify that the can is able to hold 12 fluid ounces of liquid.

Metric System: Converting Measurements of Volume

$$1 \text{ cm}^3 = 1 \text{ cc} = 1 \text{ mL}$$
$$1 \text{ cm}^3 = 1,000 \text{ mm}^3$$
$$1 \text{ m}^3 = 1,000,000 \text{ cm}^3$$
$$1 \text{ L} = 1,000 \text{ cm}^3$$
$$1 \text{ m}^3 = 1,000 \text{ L}$$

It's no surprise that the metric conversion ratios are all powers of 10.

Exercises

9. A can of Perrier mineral water has a diameter of 5.6 cm and a height of 14.7 cm. Verify that the can is able to hold 330 milliliters of liquid.

Both Systems: Converting Measurements of Volume

Converting between the U.S. and metric systems will of course involve messy decimal values. For example, because $1 \text{ in} = 2.54 \text{ cm}$, we can cube both numbers and find that $1 \text{ in}^3 = (2.54 \text{ cm})^3 \approx 16.387 \text{ cm}^3$. The conversions are rounded to three or four significant figures in the table below.

$$1 \text{ in}^3 \approx 16.39 \text{ cm}^3 \leftrightarrow 1 \text{ cm}^3 \approx 0.0612 \text{ in}^3$$
$$1 \text{ ft}^3 \approx 0.0284 \text{ m}^3 \leftrightarrow 1 \text{ m}^3 \approx 35.29 \text{ ft}^3$$
$$1 \text{ yd}^3 \approx 0.7646 \text{ m}^3 \leftrightarrow 1 \text{ m}^3 \approx 1.308 \text{ yd}^3$$

Exercises

10. A “two yard” dumpster has a volume of 2 cubic yards. Convert this to cubic meters.

11. Convert 240 in^3 to cm^3.

12. Convert 500 cm^3 to in^3.

13. Convert $1,000 \text{ ft}^3$ to m^3.

14. Convert 45 m^3 to yd^3.

Density

The density of a material is its weight per volume such as pounds per cubic foot, or mass per volume such as grams per cubic centimeter. Multiplying the volume of an object by its density will give its weight or mass.

Exercises

15. The standard size of a gold bar in the U.S. Federal Reserve is 7 inches by $3\frac{5}{8}$ inches by $1\frac{3}{4}$ inches.[1] The density of gold is 0.698 pounds per per cubic inch. How much does one gold bar weigh?

16. A cylindrical iron bar has a diameter of 3.0 centimeters and a length of 20.0 centimeters. The density of iron is 7.87 grams per cubic centimeter. What is the bar's mass, in kilograms?

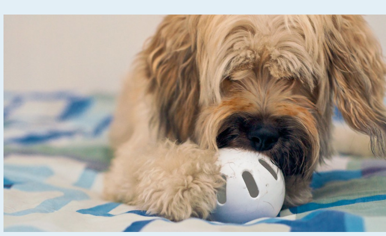
Volumes of Similar Solids

Earlier in this module, it was stated that if the linear units have a ratio of 1 to n, the cubic units will have a ratio of 1 to n^3. This applies to similar solids as well.

If the linear dimensions of two similar solids have a ratio of 1 to n, then the volumes will have a ratio of 1 to n^3.

We'll verify this in the following exercises.

Exercises

A table tennis (ping pong) ball has a diameter of 4 centimeters. A wiffle® ball has a diameter twice that of a table tennis ball.

17. Determine the volume of the wiffle® ball.

18. Determine the volume of the table tennis ball.

19. What is the ratio of the volumes of the two balls?

Rectangular solid A has dimensions 3 inches by 4 inches by 5 inches. Rectangular solid B has dimensions triple those of A's.

20. Determine the volume of the larger solid, B.

21. Determine the volume of the smaller solid, A.

22. What is the ratio of the volumes of the two solids?

Notes

1. https://www.usmint.gov/about/mint-tours-facilities/fort-knox

Module 26: Pyramids and Cones

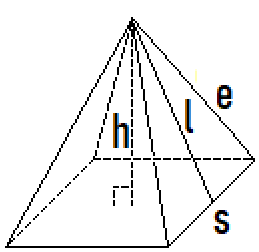
You may use a calculator throughout this module.

Note: We will not necessarily follow the rules for rounding (precision and accuracy) in this module. Many of these figures have dimensions with only one significant figure, but we would lose a lot of information if we rounded the results to only one sig fig.

In the answer key, we will often round to the nearest whole number, or to the nearest tenth, or to two or three significant figures as we deem appropriate.

Pyramids

A **pyramid** is a geometric solid with a polygon base and triangular faces with a common vertex (called the *apex* of the pyramid). Pyramids are named according to the shape of their bases. The most common pyramids have a square or another regular polygon for a base, making all of the faces identical isosceles triangles. The height, h, is the distance from the apex straight down to the center of the base. Two other measures used with pyramids are the edge length e, the sides of the triangular faces, and the slant height l, the height of the triangular faces.



Volume of a Pyramid

In general, the volume of a pyramid with base of area B and height h is

$V = \frac{1}{3}Bh$ or $V = Bh \div 3$

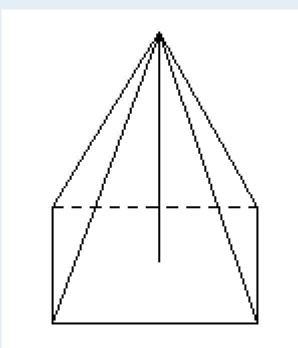
If the base is a square with side length s, the volume is

$V = \frac{1}{3}s^2h$ or $V = s^2h \div 3$

Interestingly, the volume of a pyramid is $\frac{1}{3}$ the volume of a prism with the same base and height.

Exercises

1. A pyramid has a square base with sides 16 centimeters long, and a height of 15 centimeters. Find the volume of the pyramid.



2. The Great Pyramid at Giza in Egypt has a height of 137 meters and a square base with sides 230 meters long.[1] Find the volume of the pyramid.

The lateral surface area (LSA) of a pyramid is found by adding the area of each triangular face.

Lateral Surface Area of a Pyramid

If the base of a pyramid is a regular polygon with n sides each of length s, and the slant height is l, then

$LSA = \frac{1}{2}nsl$ or $LSA = nsl \div 2$

If the base is a square, then

$LSA = 2sl$

The total surface area (TSA) is of course found by adding the area of the base B to the lateral surface area. If the base is a regular polygon, you will need to use the techniques we studied in a previous module.

Total Surface Area of a Pyramid

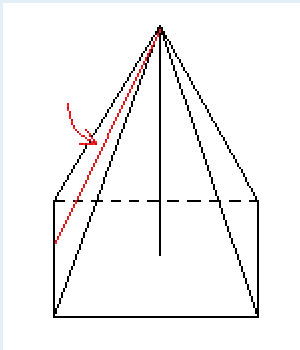
$TSA = LSA + B$

If the base is a square, then

$TSA = 2sl + s^2$

Exercises

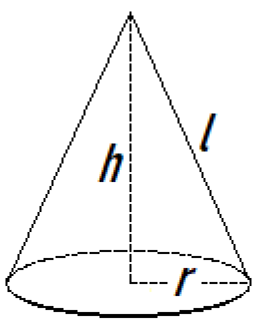
3. A pyramid has a square base with sides 16 centimeters long, and a slant height of 17 centimeters. Find the lateral surface area and total surface area of the pyramid.



4. The Great Pyramid at Giza has a slant height of 179 meters and a square base with sides 230 meters long. Find the lateral surface area of the pyramid.

Cones

A cone is like a pyramid with a circular base.



You may be able to determine the height h of a cone (the altitude from the apex, perpendicular to the base), or the slant height l (which is the length from the apex to the edge of the circular base). Note that the height, radius, and slant height form a right triangle with the slant height as the hypotenuse. We can use the Pythagorean theorem to determine the following equivalences.

The slant height$l$, height h, and radius r of a cone are related as follows:

$l = \sqrt{r^2 + h^2}$

$h = \sqrt{l^2 - r^2}$

$r = \sqrt{l^2 - h^2}$

Just as the volume of a pyramid is $\frac{1}{3}$ the volume of a prism with the same base and height, the volume of a cone is $\frac{1}{3}$ the volume of a cylinder with the same base and height.

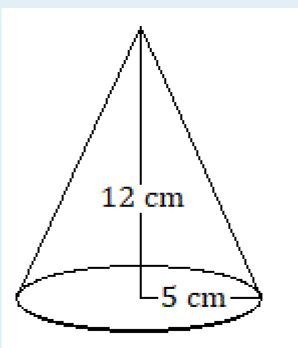
Volume of a Cone

The volume of a cone with a base radius r and height h is

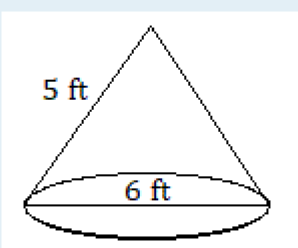
$V = \frac{1}{3}\pi r^2 h$ or $V = \pi r^2 h \div 3$

Exercises

5. The base of a cone has a radius of 5 centimeters, and the vertical height of the cone is 12 centimeters. Find the volume of the cone.



6. The base of a cone has a diameter of 6 feet, and the slant height of the cone is 5 feet. Find the volume of the cone.



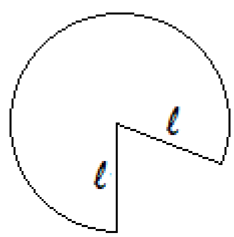
For the surface area of a cone, we have the following formulas.

Surface Area of a Cone

$LSA = \pi r l$

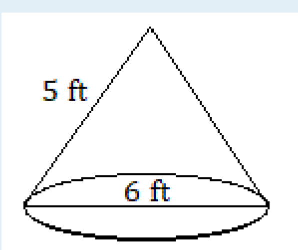
$TSA = LSA + \pi r^2 = \pi r l + \pi r^2$

It's hard to explain the justification for the LSA formula in words, but here goes. The lateral surface of a cone, when flattened out, is a circle with radius l that is missing a wedge. The circumference of this partial circle, because it matched the circumference of the circular base, is $2\pi r$. The circumference of the entire circle with radius l would be $2\pi l$, so the part we have is just a fraction of the entire circle. To be precise, the fraction is $\frac{2\pi r}{2\pi l}$, which reduces to $\frac{r}{l}$. The area of the entire circle with radius l would be πl^2. Because the partial circle is the fraction $\frac{r}{l}$ of the entire circle, the area of the partial circle is $\pi l^2 \cdot \frac{r}{l} = \pi r l$.

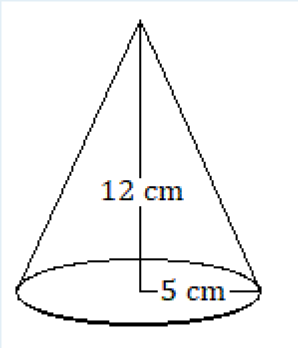


Exercises

7. The base of a cone has a diameter of 6 feet, and the slant height of the cone is 5 feet. Find the lateral surface area and total surface area of the cone.

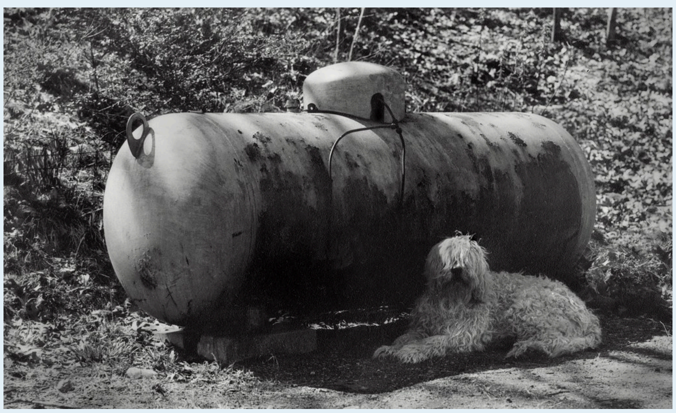


8. The base of a cone has a radius of 5 centimeters, and the vertical height of the cone is 12 centimeters. Find the lateral surface area and total surface area of the cone.



Now that we have looked at the five major solids—prism, cylinder, sphere, pyramid, cone—you should be able to handle composite solids made from these shapes. Just remember to take them in pieces.

Exercises



A 250-gallon propane tank is roughly in the shape of a cylinder with a hemisphere on each end. The length of the cylindrical part is 6 feet long, and the cross-sectional diameter of the tank is 2.5 feet.

9. Calculate the volume of the tank in cubic feet.

10. Verify that the tank can hold 250 gallons of liquid propane.

Notes

1. https://en.wikipedia.org/wiki/Great_Pyramid_of_Giza

Module 27: Percents Part 3

You may use a calculator throughout this module.

There is one more situation involving percents that often trips people up: working backwards from the result of a percent change to find the original value.

$$\text{Amount} = \text{Rate} \cdot \text{Base}$$

$$A = R \cdot B$$

Finding the Base After Percent Increase

Suppose a 12% tax is added to a price; what percent of the original is the new amount?

Well, the original number is 100% of itself, so the new amount must be $100\% + 12\% = 112\%$ of the original.

As a proportion, $\frac{A}{B} = \frac{112}{100}$. As an equation, $A = 1.12 \cdot B$.

If a number is **increased** by a percent, add that percent to 100% and use that result for R.

The most common error in solving this type of problem is applying the percent to the new number instead of the original. For example, consider this question: “After a 12% increase, the new price of a computer is \$ $1,120$. What was the original price?”

People often work this problem by finding 12% of \$ $1,120$ and subtracting that away: 12% of $1,120$ is 134.40, and $1,120 - 134.40 = 985.60$. It appears that the original price was \$ 985.60, but if we check this result, we find that the numbers don’t add up. 12% of 985.60 is 118.272, and $985.60 + 118.272 = 1,103.872$, not $1,120$.

The correct way to think about this is $1,120 = 1.12 \cdot B$. Dividing $1,120$ by 1.12 gives us the answer $1,000$, which is clearly correct because we can find that 12% of $1,000$ is 120, making the new amount $1,120$. The original price was \$ $1,000$.

To summarize, we cannot subtract 12% from the new amount; we must instead ***divide*** the new amount by 112%.

Exercises

1. A sales tax of 8% is added to the selling price of a lawn tractor, making the total price \$ $1,402.92$. What is the selling price of the lawn tractor without tax?

2. The U.S. population in 2018 was estimated to be 327.2 million, which represents a 7.6% increase from 2008. What was the U.S. population in 2008?

Finding the Base After Percent DEcrease

Suppose a 12% discount is applied to a price; what percent of the original is the new amount?

As above, the original number is 100% of itself, so the new amount must be $100\% - 12\% = 88\%$ of the original.

As a proportion, $\frac{A}{B} = \frac{88}{100}$. As an equation, $A = 0.88 \cdot B$.

> If a number is **decreased** by a percent, subtract that percent from 100% and use that result for R.

As above, the most common error in solving this type of problem is applying the percent to the new number instead of the original. For example, consider this question: "After a 12% decrease, the new price of a computer is \$ 880. What was the original price?"

People often work this problem by finding 12% of 880 and adding it on: 12% of 880 is 105.60, and $880 + 105.60 = 985.60$. It appears that the original price was \$ 985.60, but if we check this result, we find that the numbers don't add up. 12% of 985.60 is 118.272, and $985.60 - 118.272 = 867.328$, not 880.

The correct way to think about this is $880 = 0.88 \cdot B$. Dividing 880 by 0.88 gives us the answer $1,000$, which is clearly correct because we can find that 12% of $1,000$ is 120, making the new amount 880. The original price was \$ $1,000$.

To summarize, we cannot add 12% to the new amount; we must instead ***divide*** the new amount by 88%.

Exercises

3. A city department's budget was cut by 5% this year. If this year's budget is \$ 3.04 million, what was last year's budget?

4. CCC's enrollment in Summer 2019 was $9,116$ students, which was a decrease of 2.17%

from Summer 2018. What was the enrollment in Summer 2018? (Round to the nearest whole number.)[1]

5. An educational website claims that by purchasing access for \$ 5, you'll save 69% off the standard price. What was the standard price? (Use you best judgment when rounding your answer.)

Notes

1. These enrollment numbers don't match those in Percents Part 2, which makes me wonder how accurate the yearly reports are. Or maybe I inadvertently grabbed data from two different ways that enrollment was being counted.

Module 28: Mean, Median, Mode

You may use a calculator throughout this module.

We often describe data using a ***measure of central tendency***. This is a number that we use to describe the typical data value. We will now look at the ***mean***, the ***median***, and the ***mode***.

Mean

The mean of a set of data is what we commonly call the average: add up all of the numbers and then divide by how many numbers there were.

Exercises

1. The table below shows the average price of a gallon of regular unleaded gasoline in the Seattle metro area for ten weeks in Fall 2019. Compute the mean price over this time period.[1]

Sep 16, 2019	\$ 3.24
Sep 23, 2019	\$ 3.25
Sep 30, 2019	\$ 3.27
Oct 07, 2019	\$ 3.30
Oct 14, 2019	\$ 3.35
Oct 21, 2019	\$ 3.41
Oct 28, 2019	\$ 3.44
Nov 04, 2019	\$ 3.43
Nov 11, 2019	\$ 3.42
Nov 18, 2019	\$ 3.40

2. The table below shows the average price of a gallon of regular unleaded gasoline in the Seattle metro area for ten weeks in Fall 2008. Compute the mean price over this time period.[2]

Sep 15, 2008	\$ 3.77
Sep 22, 2008	\$ 3.70
Sep 29, 2008	\$ 3.65
Oct 06, 2008	\$ 3.54
Oct 13, 2008	\$ 3.36
Oct 20, 2008	\$ 3.09
Oct 27, 2008	\$ 2.78
Nov 03, 2008	\$ 2.54
Nov 10, 2008	\$ 2.38
Nov 17, 2008	\$ 2.24

Median

The median is the middle number in a set of data; it has an equal number of data values below it as above it. The numbers must be arranged in order, usually smallest to largest but largest to smallest would also work. Then we can count in from both ends of the list and find the median in the middle.

If there are an odd number of data values, there will be one number in the middle, which is the median.

If there are an even number of data values, there will be two numbers in the middle. The mean of these two numbers is the median.

Exercises

The houses on a block have these property values: \$ $250,000$; \$ $300,000$; \$ $320,000$; \$ $190,000$; \$ $220,000$.

3. Find the mean property value.

4. Find the median property value.

A new house is built on the block, making the property values \$ $250,000$; \$ $300,000$; \$ $320,000$; \$ $190,000$; \$ $220,000$ and \$ $750,000$.

5. Find the mean property value.

6. Find the median property value.

7. Which of these measures appears to give a more accurate representation of the typical house on the block?

The mean is better to work with when we do more complicated statistical analysis, but it is sensitive to extreme values; in other words, one very large or very small number can have a significant effect on the mean. The median is not sensitive to extreme values, which can make it a better measure to use when describing data that has one or two numbers very different from the remainder of the data.

For example, suppose you had ten quizzes, and you scored 100 on nine of them but missed one quiz and received a score of 0. You earned a total of 900 points over 10 quizzes, making your mean score 90. However, your median score would be 100 because the median is calculated based on your fifth and sixth highest scores.

Mode

In the example above, 100 is also the mode of your scores because it is the most common quiz score in your gradebook. The mode is the value that appears **most** frequently in the data set. On the game show *Family Feud*, the goal is to guess the mode: the most popular answer.

If no numbers are repeated, then the data set has no mode. If there are two values that are tied for most frequently occurring, then they are both considered a mode and the data set is called ***bimodal***. If there are more than two values tied for the lead, we usually say that there is no mode.[3] (It's like in sports: there is usually one MVP, but occasionally there are two co-MVPs. Having three or more MVPs would start to get ridiculous.)

Exercises

8. One hundred cell phone owners are asked which carrier they use. What is the mode of the data?

AT&T Mobility	Verizon Wireless	T-Mobile US	Dish Wireless	U.S. Cellular
43	29	24	2	2

9. Fifty people are asked what their favorite type of Girl Scout cookie is. What is the mode?

S'Mores	Samoas	Tagalongs	Trefoils	Thin Mints
4	16	5	9	16

Let's put it all together and find the mean, median, and mode of some data sets. Sportsball!

From 2001-2019, these are the numbers of games won by the New England Patriots each NFL season.[4]

year	wins
2001	11
2002	9
2003	14
2004	14
2005	10
2006	12
2007	16
2008	11
2009	10
2010	14
2011	13
2012	12
2013	12
2014	12
2015	12
2016	14
2017	13
2018	11
2019	12

Exercises

10. Find the mean number of games won from 2001 to 2019.

11. Find the median number of games won from 2001 to 2019.

12. Find the mode of the number of games won from 2001 to 2019.

13. Do any of these measures appear to be misleading, or do they all represent the data fairly well?

From 2001-2019, these are the numbers of games won by the Buffalo Bills each NFL season.[5]

year	wins
2001	3
2002	8
2003	6
2004	9
2005	5
2006	7
2007	7
2008	7
2009	6
2010	4
2011	6
2012	6
2013	6
2014	9
2015	8
2016	7
2017	9
2018	6
2019	10

Exercises

14. Find the mean number of games won from 2001 to 2019.

15. Find the median number of games won from 2001 to 2019.

16. Find the mode of the number of games won from 2001 to 2019.

17. Do any of these measures appear to be misleading, or do they all represent the data fairly well?

Some sets of data may not be easy to describe with one measure of central tendency.

Exercises

Thirteen clementines are weighed. Their masses, in grams, are
$82, 90, 90, 92, 93, 94, 94, 102, 107, 107, 108, 109, 109.$

18. Determine the mean. Does the mean appear to represent the mass of a typical clementine?

19. Determine the median. Does the median appear to represent the mass of a typical clementine?

20. Determine the mode. Does the mode appear to represent the mass of a typical clementine?

Suppose that the 108-gram clementine is a tiny bit heavier and the masses are actually
$82, 90, 90, 92, 93, 94, 94, 102, 107, 107, 109, 109, 109.$

21. Determine the new mean. Is the new mean different from the original mean?

22. Determine the new median. Is the new median different from the original median?

23. Determine the new mode. Is the new mode different from the original mode? Does it represent the mass of a typical clementine?

One of your author’s cats is named Clementine. She weighs more than 109 grams.

Notes

1. Source: https://www.eia.gov/petroleum/gasdiesel/
2. Source: https://www.eia.gov/petroleum/gasdiesel/
3. The concepts of trimodal and multimodal data exist, but we aren't going to consider anything beyond bimodal in this textbook.
4. Source: https://www.pro-football-reference.com/teams/nwe/index.htm
5. Source: https://www.pro-football-reference.com/teams/buf/index.htm

Module 29: Probability

Probability is the likelihood that some event occurs. If the event occurs, we call that a ***favorable outcome***. The set of all possible events (or outcomes) is called the ***sample space*** of the event. We will limit our focus to ***independent*** events, which do not influence each other. For example, if we roll a 5 on one die, that does not affect the probability of rolling a 5 on the other die. (We will not be studying *dependent* events, which do influence each other.)

If we are working with something simple like dice, cards, or coin flips where we know all of the possible outcomes, we can calculate the ***theoretical probability*** of an event occurring. To do this, we divide the number of ways the event can occur by the total number of possible outcomes. We may choose to write the probability as a fraction, a decimal, or a percent depending on what form seems most useful.

Theoretical probability of an event:

$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{total number of outcomes}}$$

Suppose two six-sided dice, numbered 1 through 6, are rolled. There are $6 \cdot 6 = 36$ possible outcomes in the sample space. If we are playing a game where we take the sum of the dice, the only possible outcomes are 2 through 12. However, as the following table shows, these outcomes are not all equally likely. For example, there are two different ways to roll a 3, but only one way to roll a 2.

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Exercises

Two six-sided dice numbered 1 through 6 are rolled. Find the probability of each event occuring.

1. The sum of the dice is 7.

2. The sum of the dice is 11.

3. The sum of the dice is 7 or 11.

4. The sum of the dice is greater than 1.

5. The sum of the dice is 13.

Some things to notice...

If an event is impossible, its probability is 0% or 0.

If an event is certain to happen, its probability is 100% or 1.

If it will be tedious to count up all of the favorable outcomes, it may be easier to count up the unfavorable outcomes and subtract from the total.

Exercises

Two six-sided dice numbered 1 through 6 are rolled. Find the probability of each event occuring.

6. The sum of the dice is 5.

7. The sum of the dice is not 5.

8. The sum of the dice is greater than 9.

9. The sum of the dice is 9 or lower.

The set of outcomes in which an event does not occur is called the ***complement*** of the event. The event "the sum is not 5" is the complement of "the sum is 5". Two complements *complete* the sample space.

If the probability of an event happening is p, the probability of the complement is $1 - p$.

Exercises

A bowl of 60 Tootsie Rolls Fruit Chews contains the following: 15 cherry, 14 lemon, 13 lime, 11 orange, 7 vanilla.

10. If one Tootsie Roll is randomly selected from the bowl, what is the probability that it is cherry?

11. What is the probability that a randomly selected Tootsie Roll is either lemon or lime?

12. What is the probability that a randomly selected Tootsie Roll is not orange or vanilla?

Here's where we try to condense the basics of genetic crosses into one paragraph.

Each parent gives one allele to their child. The allele for brown eyes is B, and the allele for blue eyes is b. If two parents both have genotype Bb, the table below (which biologists call a Punnett square) shows that there are four equally-likely outcomes: BB, Bb, Bb, bb. The allele for brown eyes, B, is dominant over the gene for blue eyes, b, which means that if a child has any B alleles, they will have brown eyes. The only genotype for which the child will have blue eyes is bb.

	B	*b*
B	*BB*	*Bb*
b	*Bb*	*bb*

Exercises

Two parents have genotypes *Bb* and *Bb*. (*B* = brown, *b* = blue)

13. What is the probability that their child will have blue eyes?

14. What is the probability that their child will have brown eyes?

Now suppose that one parent has genotype Bb but the other parent has genotype bb. The Punnett square will look like this.

	B	*b*
b	*Bb*	*bb*
b	*Bb*	*bb*

Exercises

Two parents have genotypes *Bb* and *bb*. (*B* = brown, *b* = blue)

15. What is the probability that their child will have blue eyes?

16. What is the probability that their child will have brown eyes?

The previous methods work when we know the total number of outcomes and we can assume that they are all equally likely. (The dice aren't loaded, for example.) However, life is usually more complicated than a game of dice or a bowl of Tootsie Rolls. In many situations, we have to observe what has happened in the past and use that data to predict what might happen in the future. If someone predicts that an Alaska Airlines flight has a 95.5% of arriving on time, that is of course based on Alaska's past rate of success.[1] When we calculate the probability this way, by observation, we call it an ***empirical probability***.

Empirical probability of an event:

$$P(\text{event}) = \frac{\text{number of favorable observations}}{\text{total number of observations}}$$

Although the wording may seem complicated, we are still just thinking about $\frac{\text{part}}{\text{whole}}$.

Exercises

A photocopier makes 250 copies, but 8 of them are unacceptable because they have toner smeared on them.

17. What is the empirical probability that a copy will be unacceptable?

18. What is the empirical probability that a copy will be acceptable?

19. Out of the next $1,000$ copies, how many should we expect to be acceptable?

An auditor examined 200 tax returns and found errors on 44 of them.

20. What percent of the tax returns contained errors?

21. How many of the next $1,000$ tax returns should we expect to contain errors?

22. What is the probability that a given tax return, chosen at random, will contain errors?

It was mentioned earlier in this module that ***independent events*** have no influence on each other. Some examples:

- Rolling two dice are independent events because the result of the first die does not affect the probability of what will happen with the second die.
- If we flip a coin ten times, each flip is independent of the previous flip because the coin doesn't remember how it landed before. The probability of heads or tails remains $\frac{1}{2}$ for each flip.
- Drawing marbles out of a bag are independent events only if we put the first marble back in the bag before drawing a second marble. If we draw two marbles at once, or we draw a second marble without replacing the first marble, these are *dependent events*, which we are not studying in this course.
- Drawing two cards from a deck of 52 cards are independent events only if we put the first card back in the deck before drawing a second card. If we draw a second card without replacing the first card, these are *dependent events*; the probabilities change because there are only 51 cards available on the second draw.

If two events are independent, then the probability of both events happening can be found by multiplying the probability of each event happening separately.

If A and B are independent events, then $P(A \text{ and } B) = P(A) \cdot P(B)$.

Note: This can be extended to three or more events. Just multiply all of the probabilities together.

Exercises

An auditor examined 200 tax returns and found errors on 44 of them.

23. What is the probability that the next two tax returns both contain errors?

24. What is the probability that the next three tax returns all contain errors?

25. What is the probability that the next tax return contains errors but the one after it does not?

26. What is the probability that the next tax return does not contain errors but the one after it does?

27. What is the probability that neither of the next two tax returns contain errors?

28. What is the probability that none of the next three tax returns contain errors?

29. What is the probability that at least one of the next three tax returns contain errors? (This one is tricky!)

Notes

1. pdf file: https://www.transportation.gov/sites/dot.gov/files/2020-08/July%202020%20ATCR.pdf

Module 30: Standard Deviation

This topic requires a leap of faith. It is one of the rare times when this textbook will say "don't worry about *why* it's true; just accept it."

A ***normal distribution***, often referred to as a ***bell curve***, is symmetrical on the left and right, with the mean, median, and mode being the value in the center. There are lots of data values near the center, then fewer and fewer as the values get further from the center. A normal distribution describes the data in many real-world situations: heights of people, weights of people, errors in measurement, scores on standardized tests (IQ, SAT, ACT)…

One of the best ways to demonstrate the normal distribution is to drop balls through a board of evenly spaced pegs, as shown here.[1] Each time a ball hits a peg, it has a fifty-fifty chance of going left or right. For most balls, the number of lefts and rights are roughly equal, and the ball lands near the center. Only a few balls have an extremely lopsided number of lefts and rights, so there are not many balls at either end. As you can see, the distribution is not perfect, but it is approximated by the normal curve drawn on the glass.

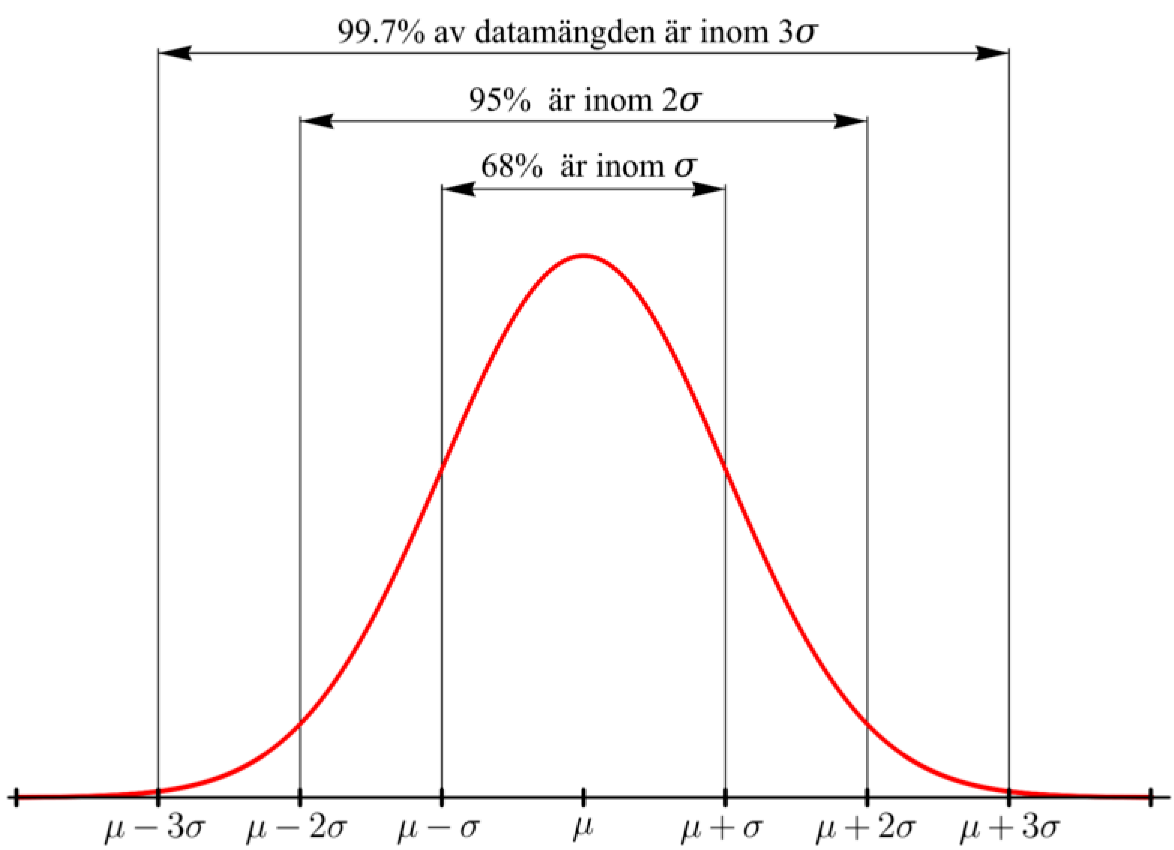
The ***standard deviation*** is a measure of the spread of the data: data with lots of numbers close to the mean has a smaller standard deviation, and data with numbers spaced further from the mean has a larger standard deviation. (In this textbook, you will be given the value of the standard deviation of the data and will never need to calculate it.) The standard deviation is a measuring stick for a particular set of data.

In a normal distribution…

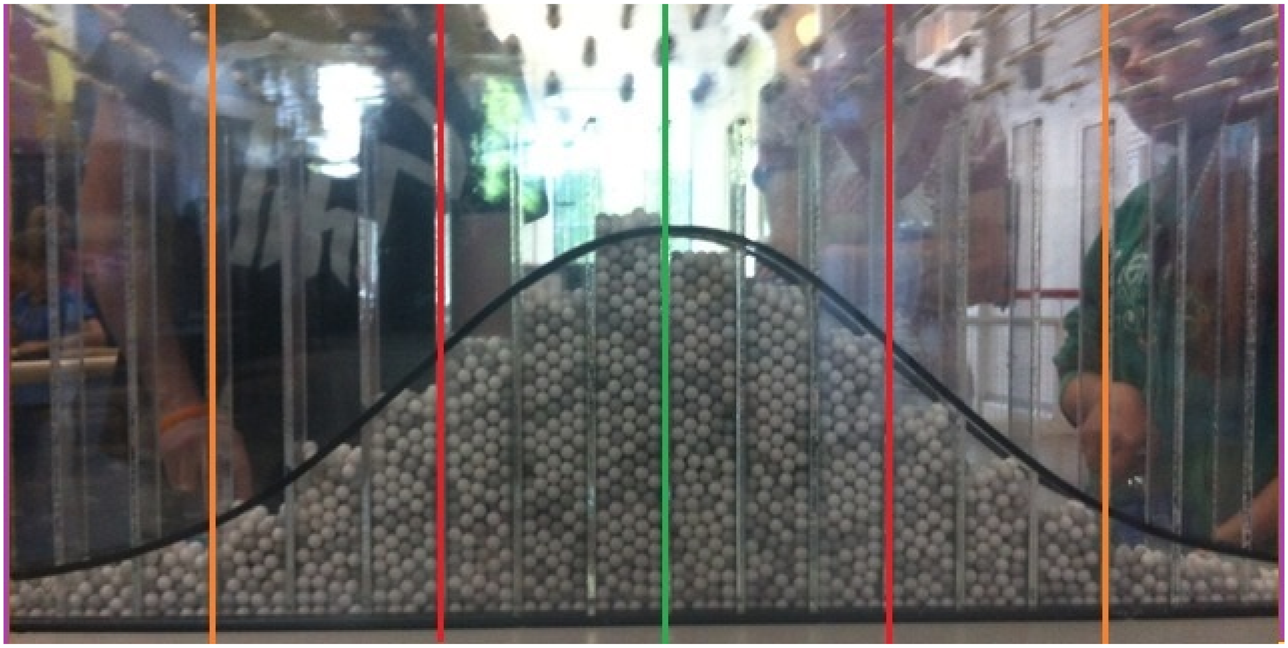
- roughly 68% of the numbers are within 1 standard deviation above or below the mean
- roughly 95% of the numbers are within 2 standard deviations above or below the mean
- roughly 99.7% of the numbers are within 3 standard deviations above or below the mean

This **68-95-99.7 rule** is called an *empirical rule* because it is based on observation rather than some formula. Nobody discovered a calculation to figure out the numbers 68%, 95%, and 99.7% until after the fact. Instead, statisticians looked at lots of different examples of normally distributed data and said "*Mon Dieu*, it appears that if you count up the data values that are within one standard deviation above or below the mean, you have about 68% of the data!" and so on.[2]

The following image is in Swedish, but you can probably decipher it because math is an international language.



Let's go back to the ball-dropping experiment, and let's assume that the standard deviation is three columns wide.[3] In the picture below, the **green** line marks the center of the distribution.



First, the two **red** lines are each three columns away from the center, which is **one standard deviation** above and below the **center**, so about **68%** of the balls will land between the red lines.

Next, the two **orange** lines are another three columns farther away from the center, which is six columns or **two standard deviations** above and below the **center**, so about **95%** of the balls will land between the orange lines.

And finally, the two **purple** lines are another three columns farther away from the center, which is nine columns or **three standard deviations** above and below the **center**, so about **99.7%** of the balls will land between the purple lines. We can expect that 997 out of $1,000$ balls will land between the purple lines, leaving only 3 out of $1,000$ landing beyond the purple lines on either end.

Here are Damian Lillard's game results for points scored, in increasing order, for the 80 games he played in the 2018-19 NBA season.[4] This is broken up into eight rows of ten numbers each, and this is a total of $2,069$ points.

$$
\begin{gathered}
11, 13, 13, 13, 14, 14, 15, 15, 15, 16,\\
16, 16, 17, 17, 17, 18, 18, 19, 19, 20,\\
20, 20, 20, 20, 21, 21, 22, 22, 23, 23,\\
23, 23, 24, 24, 24, 24, 24, 24, 24, 24,\\
25, 25, 25, 26, 26, 26, 28, 28, 28, 29,\\
29, 29, 29, 30, 30, 30, 30, 30, 31, 31,
\end{gathered}
$$

$$33, 33, 33, 33, 33, 33, 34, 34, 34, 35,$$
$$36, 36, 37, 39, 40, 40, 41, 41, 42, 51$$

Exercises

This is a review of mean, median, and mode; you'll need to know the mean in order to complete the standard deviation exercises that follow.

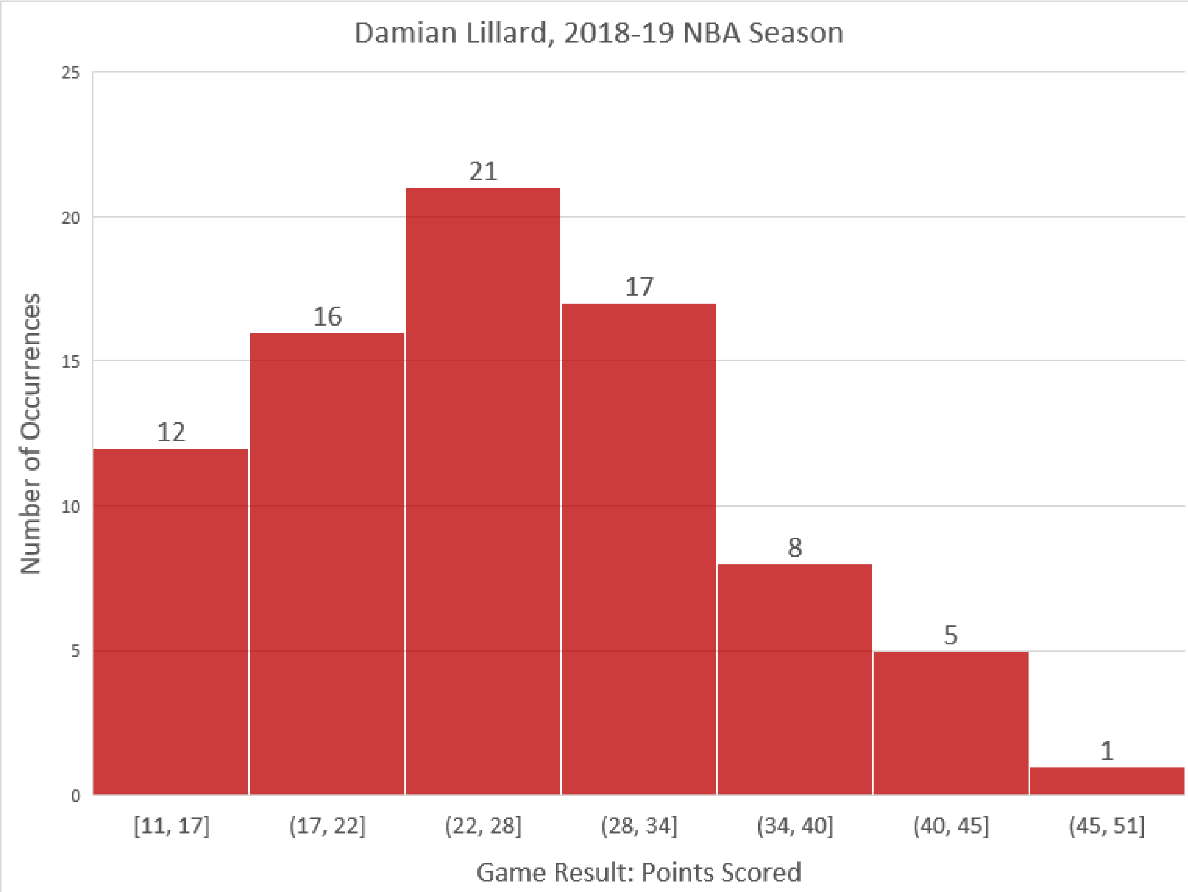
1. What is the mean of the data? (Round to the nearest tenth.)

2. What is the median of the data?

3. What is the mode of the data?

4. Do any of the mean, median, or mode seems misleading, or do all three seem to represent the data fairly well?

Here is a histogram of the data, arbitrarily grouped in seven equally-spaced intervals. It shows that the data roughly follows a bell-shaped curve, somewhat truncated on the left and with an ***outlier*** on the right.



If we enter the data into a spreadsheet program such as Microsoft Excel or Google Sheets, we can quickly find that the standard deviation is 8.2 points.

Based on the empirical rule, we should expect approximately 68% of the results to be within 8.2 points above and below the mean.

Exercises

5. Determine the range of points scored that are within one standard deviation of the mean.

6. How many of the 80 game results are within one standard deviation of the mean?

7. Is the previous answer close to 68% of the total number of game results?

And we should expect approximately 95% of the results to be within $2 \cdot 8.2 = 16.4$ points above and below the mean.

Exercises

8. Determine the range of points scored that are within two standard deviations of the mean.

9. How many of the 80 game results are within two standard deviations of the mean?

10. Is the previous answer close to 95% of the total number of game results?

And we should expect approximately 99.7% of the results to be within $3 \cdot 8.2 = 24.6$ points above and below the mean.

Exercises

11. Determine the range of points scored that are within three standard deviations of the mean.

12. How many of the 80 game results are within three standard deviations of the mean?

13. Is the previous answer close to 99.7% of the total number of game results?

Notice that we could think about the standard deviations like a measurement error or tolerance: the mean ± 8.2, the mean ± 16.4, the mean ± 24.6...

Exercises

For U.S. females, the average height is around 63.5 inches (5 ft 3.5 in) and the standard deviation is 3 inches. Use the empirical rule to fill in the blanks.

14. About 68% of the women should be between ______ and ______ inches tall.

15. About 95% of the women should be between ______ and ______ inches tall.

16. About 99.7% of the women should be between ______ and ______ inches tall.

For U.S. males, the average height is around 69.5 inches (5 ft 9.5 in) and the standard deviation is 3 inches. Use the empirical rule to fill in the blanks.

17. About 68% of the men should be between ______ and ______ inches tall.

18. About 95% of the men should be between ______ and ______ inches tall.

19. About 99.7% of the men should be between ______ and ______ inches tall.

This graph at https://tall.life/height-percentile-calculator-age-country/ shows that, because the standard deviations are equal, the two bell curves have essentially the same shape but the women's graph is centered six inches below the men's.

Exercises

Around 16% of U.S. males in their forties weigh less than 160 lb and 16% weigh more than 230 lb.[5] Assume a normal distribution.

20. What percent of U.S. males weigh between 160 lb and 230 lb?

21. What is the average weight? (Hint: think about symmetry.)

22. What is the standard deviation? (Hint: You have to work backwards to figure this out, but the math isn't complicated.)

23. Based on the empirical rule, about 95% of the men should weigh between ______ and ______ pounds.

If you are asked only one question about the empirical rule instead of three in a row (68%, 95%, 99.7%), you will most likely be asked about the 95%. This is related to the "95% confidence interval" that is often mentioned in relation to statistics. For example, the margin of error for a poll is usually close to two standard deviations.[6]

Let's finish up by comparing the performance of three NFL teams since the turn of the century.

The numbers of regular-season games won by the New England Patriots each NFL season from 2001-19:[7]

year	wins
2001	11
2002	9
2003	14
2004	14
2005	10
2006	12
2007	16
2008	11
2009	10
2010	14
2011	13
2012	12
2013	12
2014	12
2015	12
2016	14
2017	13
2018	11
2019	12

Exercises

For the Patriots, the mean number of wins is 12.2, and a spreadsheet tells us that the standard deviation is 1.7 wins.

24. There is a 95% chance of the Patriots winning between _______ and _______ games in a season.

25. In 2020, the Patriots won 7 games. Could you have predicted that based on the data? How many standard deviations from the mean is this number of wins?

The numbers of regular-season games won by the Buffalo Bills each NFL season from 2001-19:[8]

year	wins
2001	3
2002	8
2003	6
2004	9
2005	5
2006	7
2007	7
2008	7
2009	6
2010	4
2011	6
2012	6
2013	6
2014	9
2015	8
2016	7
2017	9
2018	6
2019	10

Exercises

For the Bills, the mean number of wins is 6.8, and a spreadsheet tells us that the standard deviation is 1.7 wins.

26. There is a 95% chance of the Bills winning between _______ and _______ games in a season.

27. In 2020, the Bills won 13 games. Could you have predicted that based on the data? How many standard deviations from the mean is this number of wins?

The numbers of regular-season games won by the Denver Broncos each NFL season from 2001-19:[9]

year	wins
2001	8
2002	9
2003	10
2004	10
2005	13
2006	9
2007	7
2008	8
2009	8
2010	4
2011	8
2012	13
2013	13
2014	12
2015	12
2016	9
2017	5
2018	6
2019	7

Exercises

For the Broncos, the mean number of wins is 9.1, and a spreadsheet tells us that the standard deviation is 2.6 wins.

28. There is a 95% chance of the Broncos winning between _______ and _______ games in a season.

29. In 2020, the Broncos won 5 games. Could you have predicted that based on the data? How many standard deviations from the mean is this number of wins?

Notes

1. The Plinko game on *The Price Is Right* is the best-known example of this; here's a clip of Snoop Dogg helping a contestant win some money.
2. Confession: This paragraph gives you the general idea of how these ideas developed but may not be perfectly historically accurate.
3. I eyeballed it and it seemed like a reasonable assumption.
4. Source: https://www.basketball-reference.com/players/l/lillada01/gamelog/2019
5. Source: https://www.google.com/url?sa=t&rct=j&q=&esrc=s&source=web&cd=17&ved=2ahUKEwjm-d-whavhAhWCFXwKHQxMDz4QFjAQegQIARAC&url=https%3A%2F%2Fwww2.census.gov%2Flibrary%2Fpublications%2F2010%2Fcompendia%2Fstatab%2F130ed%2Ftables%2F11s0205.pdf&usg=AOvVaw1DFDbil78g-qXbIgK6JirW
6. Source: https://en.wikipedia.org/wiki/Standard_deviation
7. Source: https://www.pro-football-reference.com/teams/nwe/index.htm
8. Source: https://www.pro-football-reference.com/teams/buf/index.htm
9. Source: https://www.pro-football-reference.com/teams/den/index.htm

Exercise Answers: Main Page

Each link below will take you to the answers to the practice exercises.

Exercise Answers: Module 1

Order of Operations

1. 7
2. 13
3. 7
4. 13
5. 2
6. 8
7. 18
8. 6
9. 25
10. 49
11. 80
12. 31
13. 28
14. 67
15. 22
16. 4
17. 160
18. 19
19. 2
20. 12

21. 40
22. 200
23. 2
24. 14
25. $9 \cdot 2 + 30 = 48^{\circ}\mathrm{F}$
26. $(72 - 30) \div 2 = 21^{\circ}\mathrm{C}$

return to main page

Exercise Answers: Module 2

Negative Numbers

1. 5
2. 5
3. -15
4. -22
5. 4
6. -4
7. -9
8. 9
9. $18^{\circ}\mathrm{F}$
10. 3
11. -200
12. 3
13. -3
14. -7
15. -7
16. 7
17. 7
18. 3
19. -3
20. $55^{\circ}\mathrm{F}$ difference

21. -12
22. -40
23. 18
24. 21
25. 4
26. -8
27. 16
28. -32
29. -7
30. -4
31. 9
32. 0
33. 0
34. undefined
35. 19
36. -73
37. 1
38. -6
39. -8
40. 40

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Exercise Answers: Module 3

Decimals

Note: When you get to Modules 5 & 6, you'll see that we would round our answers to some of exercises #3-11 differently. For now, don't worry about accuracy or precision on these exercises.

1. 90.23
2. 7.056
3. 16.55
4. 184.015
5. 8.28
6. 15.756
7. 39.15
8. 15.466
9. $\$656.25$
10. 243.5
11. $2,435$
12. $6,000$
13. $6,500$
14. $6,470$
15. 0.7
16. 0.70
17. 0.705

18. 23.4 miles per gallon

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Exercise Answers: Module 4

fractions

1. $\frac{11}{30}$
2. $\frac{19}{30}$
3. $\frac{12}{15}$
4. $\frac{8}{12}$
5. 7
6. 1
7. 0
8. undefined
9. $\frac{3}{4}$
10. $\frac{5}{3}$
11. 2
12. $\frac{1}{2}$
13. $\frac{5}{12}$
14. 1
15. at least 45 questions
16. 16
17. $\frac{3}{5}$
18. 6 scoops
19. A requires $\frac{1}{12}$ cup more than B
20. $\frac{1}{2}$ of the pizza

21. $\frac{2}{3}$ more
22. $\frac{5}{8}$ inches combined
23. $\frac{1}{8}$ inches difference
24. $\frac{7}{12}$ combined
25. $\frac{1}{12}$ more
26. 2.75
27. 0.35
28. $0.\overline{5}$ or $0.555...$
29. $1.\overline{63}$ or $1.636363...$
30. $\frac{11}{5}$
31. $\frac{20}{3}$
32. $11\frac{1}{2}$
33. $4\frac{2}{3}$
34. $10\frac{1}{12}$
35. $4\frac{7}{12}$
36. $8\frac{1}{6}$
37. $1\frac{5}{6}$ cup

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Exercise Answers: Module 5

Accuracy and Significant Figures

1. exact value
2. approximation
3. exact value
4. approximation
5. exact value
6. approximation
7. three significant figures
8. four significant figures
9. five significant figures
10. two significant figures
11. three significant figures
12. four significant figures
13. one significant figure
14. two significant figures
15. three significant figures
16. four significant figures
17. $22,000$
18. $21,800$
19. $21,840$
20. 4.3

21. 4.28
22. 4.278
23. $14,000$
24. $14,\bar{0}00$
25. $14,0\bar{0}0$
26. 2.6
27. 2.60
28. 2.600
29. $29,000$ ft
30. $29,\bar{0}00$ ft
31. $29,030$ ft
32. $29,032$ ft
33. $29,031.7$ ft
34. 107
35. 640
36. 14.4
37. 12
38. $\$ 12.90$

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Exercise Answers: Module 6

Precision and GPE

1. hundreds
2. tens
3. ones
4. thousands
5. hundreds
6. tens
7. ones
8. thousandths
9. ten thousandths
10. hundred thousandths
11. $14,000$
12. $14,\bar{0}00$
13. $14,0\bar{0}0$
14. 0.6
15. 0.60
16. 0.600
17. 39 lb
18. 39.3 lb
19. $ $9,800$
20. $ $8\bar{0}0$

21. ± 0.005 lb
22. ± 0.000005 in
23. ± 0.005 mil
24. three significant figures
25. ones place; the nearest 1 mile
26. ± 0.5 mi
27. two significant figures
28. thousands place; the nearest $1,000$ seats
29. ± 500 seats
30. three significant figures
31. tenths place; the nearest 0.1 gallon
32. ± 0.05 gal

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Exercise Answers: Module 7

Formulas

1. \$ 1.40
2. \$ 2.60
3. \$ 16.50
4. \$ 23.00
5. \$ 3.50
6. 5 representatives (rounded down from 5.43)
7. 10 representatives (rounded up from 9.57)
8. 53 representatives (rounded down from 53.14)
9. 7 electoral votes
10. 12 electoral votes
11. 55 electoral votes
12. 90 mm Hg
13. around 107 mm Hg
14. 70 in
15. 74 in
16. yes
17. no; too large
18. no; too small
19. yes
20. $50^{\circ}\mathrm{C}$

21. $37°\text{C}$
22. $-0.4°\text{F}$
23. $200°\text{C}$

return to main page

Exercise Answers: Module 8

Perimeter and Circumference

1. 36 in
2. 34 cm
3. 28 cm
4. 104 ft
5. 130 ft
6. 56 ft
7. 24 in
8. 20 cm
9. 28.3 in
10. 18.8 cm
11. 44.0 in
12. 25.1 cm

return to main page

Exercise Answers: Module 9

Percents Part 1

1. 89%
2. 11%
3. 47%
4. 53%
5. $\frac{71}{100}$
6. $\frac{1.3}{100} = \frac{13}{1000}$
7. $\frac{0.04}{100} = \frac{1}{2500}$
8. $\frac{102}{100} = \frac{51}{50}$
9. 0.71
10. 0.013
11. 0.0004
12. 1.02
13. 23%
14. 7%
15. 8.5%
16. 250%
17. 28%
18. 12.5%
19. 31.5
20. 22.5

21. 67.5
22. 100
23. 38.6
24. 2.25
25. $\$ 9.35$
26. $\$ 119.32$

return to main page

Exercise Answers: Module 10

Ratios, Rates, Proportions

1. $\frac{3}{8}$
2. $\frac{105 \text{ mi}}{2 \text{ hr}}$
3. $\frac{52.5 \text{ mi}}{1 \text{ hr}}$ or 52.5 miles per hour
4. \$ 0.199/oz, or around 20 cents per ounce
5. \$ 0.249/oz, or around 25 cents per ounce
6. the 18-ounce box has the lower unit price
7. $\frac{3}{4} = \frac{3}{4}$; true
8. $\frac{2}{3} \neq \frac{4}{5}$; false
9. $168 = 168$; true
10. $200 \neq 240$; false
11. $70 \neq 60$; false
12. $20 = 20$; true
13. $x = 12$
14. $n = 5$
15. $k = 4$
16. $w = 10$
17. $x = 10.4$
18. $m = 2.0$
19. 256 miles
20. 20 hours

21. 190 pixels wide

return to main page

Exercise Answers: Module 11

Scientific Notation

1. Earth's mass is larger because it's a 25-digit number and Mars' mass is a 24-digit number, but it might take a lot of work counting the zeros to be sure.
2. Earth's mass is about ten times larger, because the power of 10 is 1 higher than that of Mars.
3. A chlorine atom's radius is larger because it has 9 zeros before the significant digits begin, but a hydrogen atom's radius has 10 zeros before the significant digits begin. As above, counting the zeros is a pain in the neck.
4. The chlorine atom has a larger radius because its power of 10 is 1 higher than that of the hydrogen atom. (Remember that -10 is larger than -11 because -10 is farther to the right on a number line.)
5. 1.234×10^3
6. 1.02×10^7
7. 8.7×10^{-4}
8. 7.32×10^{-2}
9. $35,000$
10. $90,120,000$
11. 0.00825
12. 0.000014
13. 8×10^7
14. 3.5×10^{13}

15. 6×10^{-5}
16. 4.8×10^{5}
17. the proton's mass is roughly $1,830$ or 1.83×10^{3} times larger
18. 1.67×10^{-21} kg
19. 1.67×10^{-18} kg
20. 330.2×10^{6}; 3.302×10^{8}
21. 7.68×10^{9}; 7.68×10^{9}
22. 26.6×10^{12}; 2.66×10^{13}

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Exercise Answers: Module 12

Percents Part 2 and Error Analysis

1. 93% or 93.3%
2. 37.5%
3. \$ $2,500$
4. 720
5. 93% or 93.3%
6. 37.5%
7. \$ $2,500$
8. 720
9. \$ 19.35
10. 44.8% or 45%
11. 400 people were surveyed
12. 56.2% increase
13. 12.2% increase
14. 36.0% decrease
15. 35.7% decrease
16. $0.1875 \div 25 \approx 0.8\%$
17. $0.13 \div 10.8 \approx 1.2\%$
18. 4.806 g; 5.194 g
19. $0.21 \div 5.000 = 4.2\%$
20. 5.443 g; 5.897 g

21. $0.24 \div 5.670 \approx 4.2\%$

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Exercise Answers: Module 13

The US Measurement System

We generally won't worry about significant figures in these answers; we'll probably say "2 miles" even if "2.000 miles" is technically correct.

1. 54 in
2. 54 ft
3. 36 in
4. $1,760$ yd
5. $14\frac{2}{3}$ ft or 14 ft 8 in
6. 15 yd
7. 2 mi
8. 30 yd
9. 40 oz
10. $2,400$ lb
11. 18.75 lb
12. $32,000$ oz
13. 48 fl oz
14. 7 pt
15. 8 pt
16. 5 c
17. 1.25 gal
18. 64 fl oz

19. 3 lb 7 oz
20. 7 c 3 fl oz
21. 15 ft 2 in
22. 4 t 500 lb
23. 20 lb or 20 lb 0 oz combined
24. 3 lb 2 oz heavier
25. 9 ft 1 in combined
26. 1 ft 5 in longer

return to main page

Exercise Answers: Module 14

The Metric System

1. 5 m
2. 28 cm
3. 3.8 km
4. 1.6 m
5. 160 cm
6. 3 mm
7. 370 cm
8. $3,700 \text{ mm}$
9. $2,450 \text{ m}$
10. $245,000 \text{ cm}$
11. 0.342 m
12. 34.2 cm
13. 0.528 km
14. 0.45 m
15. 100 g
16. 80 kg
17. 500 mg
18. $2,000 \text{ kg}$
19. 81.3 cg
20. 813 g

21. $1,250$ g
22. $1,250,000$ mg
23. 0.96 g
24. 96 cg
25. 135 dag
26. 0.075 g
27. 50 L
28. 30 mL
29. $2,800$ mL
30. 28 dL
31. 150 L
32. 75 mL
33. 60 cL
34. 0.6 L
35. 0.45 L
36. 5.5 L
37. they are equal in size
38. 4 bottles
39. about 11 to 1

return to main page

Exercise Answers: Module 15

Converting Between Systems

We will generally round these answers to three significant figures; your answer may be slightly different depending on which conversion ratio you used.

1. 15.2 cm
2. 183 m
3. 366 cm
4. 164 ft
5. 16.4 yd
6. 29.5 in
7. not exactly; the error is around 0.3%.
8. they are essentially the same; the error is around 0.015%.
9. 113 g
10. 54.5 kg
11. 1.76 oz
12. 11.0 lb
13. 227 g
14. about $ 1.09
15. about $ 0.96
16. 13.3 L
17. 355 mL
18. 1.69 fl oz

19. 0.53 gal
20. a bit less than 600 km
21. 11 km/L

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Exercise Answers: Module 16

Other Conversions

We will generally round these answers to three significant figures; your answer may be slightly different depending on which conversion ratio you used.

1. $525,600$ min
2. this is roughly 31.7 years, which is indeed possible
3. 37.6 km/hr
4. 23.3 mi/hr
5. $1,770$ mi/hr
6. 29.5 mi in 1 min
7. 20.3 min
8. 0.17 mi/gal
9. 5.8 gal/mi
10. 171 gal in 1 min
11. the capacity increased by a factor of 14.4
12. 4 times greater
13. $1,200$ megawatts per home
14. 1 watt per gallon
15. $2,500$ times more powerful
16. 0.4 ms, 0.04 ms, 0.004 ms; 400 µs, 40 µs, 4 µs
17. the ratio of the wavelengths of red and infrared is 7 to 100;
    the ratio of the wavelengths of infrared and red is around 14 to 1

18. this is equivalent to $2,500$ chest x-rays

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Exercise Answers: Module 17

Angles

1. right angle
2. obtuse angle
3. reflexive angle
4. straight angle
5. acute angle
6. $a = 123°$; $b = 57°$; $c = 123°$
7. $37°$
8. $97°$
9. $23°$ each
10. $45°$ each
11. $60°$ each
12. $A = 61°$; $B = 80°$; $C = 39°$
13. $67.815°$
14. $19.6236°$
15. $34.2367°$
16. $26°47'6''$
17. $58°12'57.6''$
18. $41°7'48''$

return to main page

Exercise Answers: Module 18

Triangles

1. right isosceles triangle
2. obtuse scalene triangle
3. acute equilateral triangle (yes, an equilateral triangle will always be acute)
4. $w = 35$ ft
5. $x = 8$ cm; $y = 10.5$ cm
6. $d = 268$ ft
7. $n = 55$ cm
8. right triangle, because $5^2 + 12^2 = 13^2$
9. not a right triangle, because $8^2 + 17^2 \neq 19^2$
10. 7.07
11. 17.20
12. 30.71
13. 10 ft
14. 15 ft
15. 12.3 cm
16. 1.8 cm

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Exercise Answers: Module 19

Area of Polygons and Circles

We may occasionally violate the rounding rules in these answers so that you can be sure that your answer matches ours.

1. 20 cm^2
2. 16 cm^2
3. 4.86 m^2
4. 12.25 ft^2
5. 120 in^2
6. 360 m^2
7. 210 ft^2
8. 126 cm^2
9. 38.5 cm^2
10. 204 ft^2
11. 36 m^2
12. 124 cm^2
13. 288 cm^2
14. 28.3 cm^2
15. 50.3 cm^2
16. 153.9 in^2
17. 63.6 in^2
18. 19.6 m^2

19. $28.9\ \mathrm{ft}^2$

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Exercise Answers: Module 20

Composite Figures

1. 64 ft
2. 189 ft^2
3. 74 m
4. 235 m^2
5. Based on the stated measurements, the distance around the track will be 401 meters, which appears to be 1 meter too long. In real life, precision would be very important here, and you might ask for the measurements to be given to the nearest tenth of a meter.
6. around $9,620 \text{ m}^2$
7. around 21.5% (Hint: Make up an easy number for the side of the square, like 2 or 10.)

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Exercise Answers: Module 21

Converting Units of Area

We may occasionally violate the rounding rules in these answers so that you can be sure that your answer matches ours.

1. $43,560 \text{ ft}^2$
2. $4,840\text{yd}^2$
3. 144 in^2
4. 162 ft^2
5. 8.3 ac
6. $180,000 \text{ cm}^2$
7. 4 ha
8. 376 km^2
9. $2,000 \text{ ha}$ (to one sig fig) or $2,\overline{0}00 \text{ ha}$ (to two sig figs)
10. $6\overline{0}0 \text{ cm}^2$ (to two sig figs)
11. $75,300 \text{ ft}^2$
12. 154 in^2
13. 38.5 in^2
14. 4 to 1
15. 54 cm^2
16. 6 cm^2
17. 9 to 1

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Exercise Answers: Module 22

Surface Area of Common Solids

1. 36 cm^2
2. 76 cm^2
3. 471 cm^2
4. 628 cm^2
5. 616 cm^2
6. 380 cm^2

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Exercise Answers: Module 23

Area of Regular Polygons

1. $6,900 \text{ in}^2$
2. 94 cm^2
3. 750 in^2
4. 750 mm^2
5. 480 cm^2
6. 110 m^2
7. 280 mm^2 (the area of the circle $\approx 1,660 \text{ mm}^2$ and the area of the hexagon is $1,380 \text{ mm}^2$)

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Exercise Answers: Module 24

Volume of Common Solids

1. 40 cm^3
2. 531 cm^3
3. 45 cm^3
4. 350 cm^3
5. 520 cm^3
6. $22,600 \text{ ft}^3$
7. 3.7 mm^3
8. $1,440 \text{ cm}^3$
9. $7\bar{0}0 \text{ in}^3$
10. 31.2 cm^3 (the cylinder's volume $\approx 14.07 \text{ cm}^3$ and the hemisphere's volume $\approx 17.16 \text{ cm}^3$.)

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Exercise Answers: Module 25

Converting Units of Volume

1. $1,728 \text{ in}^3$
2. $46,656 \text{ in}^3$
3. $1,000 \text{ mm}^3$
4. $1,000,000 \text{ cm}^3$
5. the result is very close to 1 cubic yard:
   $(112 \text{ in} \cdot 14 \text{ in} \cdot 10 \text{ in}) \cdot 3 \text{ crates} = 47,040 \text{ in}^3 \approx 1.01 \text{ yd}^3$
6. this estimate is also 1 cubic yard: $(9 \text{ ft} \cdot 1 \text{ ft} \cdot 1 \text{ ft}) \cdot 3 \text{ crates} = 27 \text{ ft}^3 = 1 \text{ yd}^3$
7. around 60 gallons
8. yes, the can is able to hold 12 fluid ounces; the can's volume is roughly 23.3 cubic inches ≈ 12.9 fluid ounces.
9. yes, the can is able to hold 330 milliliters; the can's volume is roughly 362 cubic centimeters, which is equivalent to 362 milliliters.
10. 1.53 m^3
11. $3,930 \text{ cm}^3$
12. 30.5 in^3
13. 28.3 m^3
14. 59 yd^3
15. 31 lb
16. 1.1 kg
17. 268 cm^3

18. 33.5 cm^3
19. 8 to 1
20. $1,620 \text{ in}^3$
21. 60 in^3
22. 27 to 1

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Exercise Answers: Module 26

Pyramids and Cones

1. $1,280 \text{ cm}^3$
2. 2.4 million m^3
3. 544 cm^2; $80\overline{0} \text{ cm}^2$
4. $82,300 \text{ m}^2$; $135,000 \text{ m}^2$
5. 314 cm^3
6. 37.7 ft^3
7. 47.1 ft^2; 75.4 ft^2
8. 204 cm^2; 283 cm^2
9. 37.6 ft^3 (the cylinder's volume $\approx 29.45 \text{ ft}^3$ and the two hemispheres' combined volume $\approx 8.18 \text{ ft}^3$)
10. $37.6 \text{ ft}^3 \approx 281.5 \text{ gal}$, which is more than 250 gal.

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Exercise Answers: Module 27

Percents Part 3

1. \$ $1,299.00$
2. 304.1 million
3. \$ 3.20 million
4. $9,318$ students
5. \$ 16.13, or more likely, just \$ 16.

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Exercise Answers: Module 28

Mean, Median, Mode

1. $ 3.35
2. $ 3.11
3. $ $256,000$
4. $ $250,000$
5. $ $338,000$
6. $ $275,000$
7. the median is more representative because the mean is higher than five of the six home values.
8. AT&T Mobility
9. Samoas and Thin Mints
10. 12.2 games
11. 12 games
12. 12 games
13. they all represent the data fairly well; 12 wins represents a typical Patriots season.
14. 6.8 games
15. 7 games
16. 6 games
17. they all represent the data fairly well; 6 or 7 wins represents a typical Bills season.

18. 98.2 grams; the mean doesn't seem to represent a typical clementine because there is a group of smaller ones (from 82 to 94 grams) and a group of larger ones (from 102 to 109 grams) with none in the middle.
19. 94 grams; for the same reason, the median doesn't represent a typical clementine, but you could say it helps split the clementines into a lighter group and a heavier group.
20. there is no mode because too many values appear twice.
21. 98.3 grams; this is a small increase over the previous mean.
22. 94 grams; the median does not change when one of the highest numbers increases.
23. 109 grams; you might say it represents the mass of a typical large clementine, but it doesn't represent the entire group.

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Exercise Answers: Module 29

Probability

1. $\frac{6}{36} = \frac{1}{6}$
2. $\frac{2}{36} = \frac{1}{18}$
3. $\frac{8}{36} = \frac{2}{9}$
4. $\frac{36}{36} = 1$
5. $\frac{0}{36} = 0$
6. $\frac{4}{36} = \frac{1}{9}$
7. $\frac{32}{36} = \frac{8}{9}$
8. $\frac{6}{36} = \frac{1}{6}$
9. $\frac{30}{36} = \frac{5}{6}$
10. $\frac{15}{60} = \frac{1}{4}$
11. $\frac{27}{60} = \frac{9}{20}$
12. $\frac{42}{60} = \frac{7}{10}$
13. $\frac{1}{4} = 25\%$
14. $\frac{3}{4} = 75\%$
15. $\frac{2}{4} = 50\%$
16. $\frac{2}{4} = 50\%$
17. $\frac{8}{250} = 3.2\%$
18. $\frac{242}{250} = 96.8\%$
19. we should expect 968 copies to be acceptable
20. $\frac{44}{200} = 22\%$

21. we should expect 220 tax returns to have errors
22. $22\% = 0.22$
23. $(0.22)^2 \approx 4.8\%$
24. $(0.22)^3 \approx 1.1\%$
25. $0.22 \cdot 0.78 \approx 17.2\%$
26. $0.78 \cdot 0.22 \approx 17.2\%$
27. $(0.78)^2 \approx 60.8\%$
28. $(0.78)^3 \approx 47.5\%$
29. $1 - (0.78)^3 \approx 52.5\%$

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Exercise Answers: Module 30

Standard Deviation

1. 25.9 points
2. 24.5 points
3. 24 points (which occurred eight times)
4. all three seem to represent the typical number of points scored; the mean is a bit high because there are no extremely low values but there are a few high values that pull the mean upwards.
5. 17.7 to 34.1 points
6. 54 of the 80 game results
7. yes; $54 \div 80 = 67.5\%$
8. 9.5 to 42.3 points
9. 79 of the 80 game results
10. sort of close but not really; $79 \div 80 = 98.75\%$
11. 1.3 to 50.5 points
12. 79 of the 80 game results, again
13. yes, this is pretty close; $79 \div 80 = 98.75\%$
14. 60.5; 66.5
15. 57.5; 69.5
16. 54.5; 72.5
17. 66.5; 72.5
18. 63.5; 75.5

19. 60.5; 78.5
20. 68% because $100\% - (16\% + 16\%) = 68\%$
21. 195 lb because this is halfway between 160 and 230 lb
22. 35 lb because $195 - 35$ lb and $195 + 35$ lb encompasses 68% of the data
23. 125; 265
24. 8.8; 15.6
25. You would not have predicted this from the data because it is more than two standard deviations below the mean, so there would be a roughly 2.5% chance of this happening randomly. In fact, $(12.2 - 7) \div 1.7$ is slightly larger than 3, so this is more than three standard deviations below the mean, making it even more unlikely. (You might have predicted that the Patriots would get worse when Tom Brady left them for Tampa Bay, but you wouldn't have predicted only 7 wins based on the previous nineteen years of data.)
26. 3.4; 10.2
27. You would not predict this from the data because it is more than two standard deviations above the mean, so there would be a roughly 2.5% chance of this happening randomly. In fact, $(13 - 6.8) \div 1.7 \approx 3.6$, so this is more than three standard deviations above the mean, making it even more unlikely. This increased win total is partly due to external forces (i.e., the Patriots becoming weaker and losing two games to the Bills) but even 11 wins would have been a bold prediction, let alone 13.
28. 3.9; 14.3
29. The trouble with making predictions about the Broncos is that their standard deviation is so large. You could choose any number between 4 and 14 wins and be within the 95% interval. $(9.1 - 5) \div 2.6 \approx 1.6$, so this is around 1.6 standard deviations below the mean, which makes it not very unusual. Whereas the Patriots and Bills are more consistent, the Broncos' win totals fluctuate quite a bit and are therefore more unpredictable.

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